PROCEEDINGS

OF THE

AMERICAN SOCIETY OF CIVIL ENGINEERS

VOL. 68 — OCTOBER, 1942 — No. 8, PART 1

TECHNICAL PAPERS

AND

DISCUSSIONS

Published monthly, except July and August, at Prince and Lemon Streets, Lancaster, Pa., by the American Society of Civil Engineers. Editorial and General Offices at 33 West Thirty-ninth Street, New York, N. Y. Reprints from this publication may be made on condition that the full title of Paper, name of Author, page reference, and date of publication by the Society, are given.

Entered as Second-Class Matter, September 23, 1937, at the Post Office at Lancaster, Pa., under the Act of March 3, 1879. Acceptance for mailing at special rate of postage provided for in Section 1103, Act of October 3, 1917, authorized on July 5, 1918.

Subscription (if entered before January 1) $8.00 per annum. Price $1.00 per copy

CURRENT PAPERS AND DISCUSSIONS

		Discussion closes
Fundamental Aspects of the Depreciation Problem: A Symposium	Nov., 1941	
Discussion	Nov., Dec., 1941, Feb., Mar., Apr., June, Sept., 1942	Closed*
Stability of Granular Materials. *R. E. Glover and F. E. Cornwell*	Nov., 1941	
Discussion	Nov., 1941, Mar., Apr., Oct., 1942	Closed
Timber Friction Pile Foundations. *Frank M. Masters*	Nov., 1941	
Discussion	Feb., Apr., May, June, Sept., Oct., 1942	Closed
Bending Moments in the Walls of Rectangular Tanks. *Dana Young*	Nov., 1941	
Discussion	Feb., Sept., 1942	Closed*
Energy Loss at the Base of a Free Overfall. *Walter L. Moore*	Nov., 1941	
Discussion	Jan., Apr., June, 1942	Closed*
Development of Transportation in the United States. *J. E. Teal*	Dec., 1941	
Discussion	Feb., Mar., Apr., May, 1942	Closed*
Lateral Stability of Unsymmetrical I-Beams and Trusses in Bending. *George Winter*	Dec., 1941	
Discussion	Feb., Mar., Oct., 1942	Closed
Drainage of Leveed Areas in Mountainous Valleys. *Gordon R. Williams*	Jan., 1942	
Discussion	Jan., Apr., May, June, 1942	Closed*
Design of Sign Letter Sizes. *Adolphus Mitchell and T. W. Forbes*	Jan., 1942	
Discussion	May, June, 1942	Closed*
Hydraulic Design of Drop Structures for Gully Control. *B. T. Morris and D. C. Johnson*	Jan., 1942	
Discussion	Apr., May, June, Sept., Oct., 1942	Closed*
Analytical and Experimental Methods in Engineering Seismology. *M. A. Biot*	Jan., 1942	
Discussion	Apr., May, June, Sept., 1942	Closed*
Reduction of Mineral Content in Water with Organic Zeolites. *R. F. Goudey*	Feb., 1942	
Discussion	Feb., Apr., May, 1942	Closed*
Hydrodynamics of Model Storm Sewer Inlets Applied to Design. *G. S. Tapley*	Mar., 1942	
Discussion	Sept., 1942	Closed*
Basic Economic Considerations Affecting Single and Multiple Purpose Projects *Otis M. Page*	Mar., 1942	
Discussion	Sept., 1942	Closed*
Design of St. Georges Tied Arch Span. *J. M. Garrelts*	Dec., 1941	
Discussion	Feb., Apr., May, June, Sept., Oct., 1942	Nov. 1, 1942
Allocation of the Tennessee Valley Authority Projects. *Theorore B. Parker*	Dec., 1941	
Discussion	Feb., Mar., May, June, 1942	Nov. 1, 1942
Effect of Variation of Elastic Characteristics on Static Unknowns. *A. Hrennikoff*	Jan., 1942	
Discussion	Apr., May, Oct., 1942	Nov. 1, 1942
The Grease Problem in Sewage Treatment. *Almon L. Fales and Samuel A. Greeley*	Feb., 1942	
Discussion	Feb., May, June, Sept., 1942	Nov. 1, 1942
Unusual Events and Their Relation to Federal Water Policies. *W. G. Hoyt*	Feb., 1942	
Discussion	Apr., Sept., 1942	Nov. 1, 1942
Resistance of Soft Fill to Static Wheel Loads. *W. Watters Pagon*	Feb., 1942	Nov. 1, 1942
Viscosity and Surface Tension Effects on V-Notch Weir Coefficients. *Arno T. Lenz*	Mar., 1942	
Discussion	June, Sept., 1942	Nov. 1, 1942
Early Contributions to Mississippi River Hydrology. *C. S. Jarvis*	Mar., 1942	
Discussion	Sept., 1942	Nov. 1, 1942
Stress Concentrations in Plates Loaded Over Small Areas. *H. M. Westergaard*	Apr., 1942	
Discussion	Sept., Oct., 1942	Nov. 1, 1942
Profile Curves for Open-Channel Flow. *Dwight F. Gunder*	Apr., 1942	
Discussion	June, Sept., Oct., 1942	Nov. 1, 1942
Model Tests on Structures for Hydroelectric Developments. *L. M. Davis*	Apr., 1942	
Discussion	Apr., June, 1942	Nov. 1, 1942
Design of Structural Members: First Progress Report of the Committee of the Structural Division on Design of Structural Members	Apr., 1942	Nov. 1, 1942
Classification of Irrigable Lands. *W. W. Johnston*	May, 1942	
Discussion	May, Oct., 1942	Dec. 1, 1942
Moment Balance: A Self-Checking Analysis of Rigidly Jointed Frames. *R. J. Cornish*	May, 1942	
Discussion	May, June, Sept., Oct., 1942	Dec. 1, 1942
Numerical Procedure for Computing Deflections, Moments, and Buckling Loads. *N. M. Newmark*	May, 1942	
Discussion	May, June, Sept., Oct., 1942	Dec. 1, 1942
Mechanics of Creep for Structural Analysis. *Joseph Marin*	May, 1942	
Discussion	Sept., 1942	Dec. 1, 1942
Relative Angular, Linear, and Traverse Accuracies in City Surveys. *S. A. Bauer*	May, 1942	
Discussion	Oct., 1942	Dec. 1, 1942
Physical Properties of Driven and Undriven Rivets of High-Strength Structural Steels: Progress Report of the Committee of the Structural Division on Structural Alloys	May, 1942	Dec. 1, 1942
Earth Pressure and Shearing Resistance of Plastic Clay: A Symposium	June, 1942	
Discussion	Sept., 1942	Dec. 1, 1942
Protective and Remedial Measures for Sanitary and Public Health Engineering Services: Second Progress Report of the Sanitary and Public Health Engineering Division of the National Committee of the Society, on Civilian Protection in War Time	June, 1942	
Advancement of Sanitary Engineering: Report of the Committee of the Sanitary Engineering Division on Advancement of Sanitary Engineering	June, 1942	
Statistical Analysis in Hydrology. *L. R. Beard*	Sept., 1942	
Discussion	Oct., 1942	Feb. 1, 1943
Permeability of Mud Mountain Dam Core Material. *Allen S. Cary, Boyd H. Walter and Howard T. Harstad*	Sept., 1942	Feb. 1, 1943
Entrainment of Air in Flowing Water: A Symposium	Sept., 1942	Feb. 1, 1943

NOTE.—The closing dates herein published are final except when names of prospective discussers are registered for special extension of time.

* Publication of closing discussion pending.

CONTENTS FOR OCTOBER, 1942

PAPERS

DISCUSSIONS

CONTENTS FOR OCTOBER, 1942 (*Continued*)

For Index to all Papers, the discussion of which is current in PROCEEDINGS, *see page 2*

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

CONFORMITY BETWEEN MODEL AND PROTOTYPE

A SYMPOSIUM

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **March 1, 1943**.

FOREWORD

A comprehensive Symposium on the conformity of model to prototype behavior has been in the minds of the Executive Committee of the Hydraulics Division for a number of years. The Committee on Hydraulic Research (J. C. Stevens, M. Am. Soc. C. E., Chairman), appointed in April, 1934, has announced this subject as one of its major interests (1),[1] and the present Symposium is the result of the efforts of Subcommittee 2, comprising Clarence E. Bardsley, M. Am. Soc. C. E., and Joseph B. Tiffany, Jr., Jun. Am. Soc. C. E., with E. W. Lane, M. Am. Soc. C. E., as Chairman. Credit should also be given to Paul W. Thompson, Assoc. M. Am. Soc. C. E., and Herbert D. Vogel, M. Am. Soc. C. E. Colonel Thompson succeeded Major Vogel on the committee and was in turn succeeded by Mr. Tiffany.

Two of the papers were first presented at the San Diego, Calif., meeting of the Division in July, 1941; one was presented at the Denver, Colo., meeting of the Division in July, 1940; the remaining papers have not been presented publicly in any form. Some papers on this subject have been published separately and these are included in the Bibliography (2) (3) (24). Each author is outstanding in the art of model experimentation and in the analysis of results, and has had an opportunity to determine how nearly the model tests conformed to the prototype observations.

The objective of this research can be stated as a single question: "Does the prototype act as the model predicted?" It is rather difficult to obtain complete conformity for two or three reasons. For example, working conditions may not even approach the load limit that was used in the model; and the prototype rarely is built precisely in accord with the model. The purpose of the Symposium is to assemble the results of a wide variety of studies in which actual comparisons have been made so that hydraulic engineers may judge the sufficiency of the model as a tool in design.

[1] Numerals in parentheses, thus: (1), refer to corresponding items in the Bibliography, which is appended to the Symposium as a separate unit.

HYDRAULIC STRUCTURES

By Jacob E. Warnock,[2] M. Am. Soc. C. E., and H. G. Dewey, Jr.,[3] Jun. Am. Soc. C. E.

Synopsis

Some of the general factors involved in making model-prototype comparisons of hydraulic structures are mentioned in this paper. Reasons for desiring comparisons, the types made, the difficulties encountered, and the measurements and instruments required are discussed briefly. Citing two of the comparisons made by the Bureau of Reclamation as examples, an account is given of a small irrigation structure, and of the pressure and discharge tests on the 102-in. river outlets of a dam in a western state.

Introduction

Although great progress was made during the 1932–1942 decade in the use of hydraulic models to aid in the design of hydraulic structures, there has been but little progress in comparing the performance of a model and its prototype. It is of considerable interest and importance to make such a comparison in order to determine the degree with which the model may be relied upon for quantitative results even if the laws of similitude have been rigidly followed. In the field of hydraulic structures this is of particular value when applied to discharge coefficients, water-surface profiles, erosion of river beds, and pressures on boundary surfaces, among others. Of nearly equal importance is the fact that reliable prototype measurements will furnish accurate design data enabling more economical designs in the future.

The reasons why little progress has been made in comparing the hydraulic performance of a model and its prototype may be explained by a lack of interest, particularly after the prototype is in satisfactory operation; a natural reliance on model theory; and the difficulties of making prototype measurements. Fortunately, considerable interest is now being shown for making these comparisons and it is believed that this Symposium not only will assist materially in helping to create more interest but, perhaps, from the exchange of ideas, the difficulties involved in making the comparison will be lessened.

Accordingly, this paper will treat some of the recognized difficulties in procuring prototype measurements, together with the measurements and instruments required. Two types of comparisons between model and prototype will follow, one qualitative and the other quantitative. The former is the easiest and the most frequently made. It consists of describing, for example, the observed performance of some structure for several flows, and then comparing these observations with those made of similar flows in the model. It is recog-

[2] Engr., Hydr. Testing Laboratory, U. S. Bureau of Reclamation, Denver, Colo.

[3] 1st Lt., U. S. Army, U. S. Waterways Experiment Station, Vicksburg, Miss.

nized that this type of comparison is of particular value only to those most familiar with the structure, is not too convincing to others, and leads the engineer no nearer to actually verifying the quantitative results of the model. Obviously, a quantitative comparison is the most valuable since it not only will permit direct comparison between several quantitative factors of the model and prototype, but it will also begin to establish the degree with which the model can be relied upon. The hydraulic model is reliable to a high degree, because the laws of similitude in themselves are fundamentally sound when used correctly; but it is of importance to obtain concrete evidence and thereby recognize the influence of any digressions made in conducting the model experiments.

Factors Making Comparisons Difficult

Prototype Measurements.—The difficulty of obtaining accurate prototype data is the principal reason why correlation work has been delayed. Although several methods are available for measuring the flow, the difficulty lies not only in actually being able to procure the measurements, but also in being able to obtain data applicable to the particular feature of the prototype for which a comparison is to be made. In these respects the discharge, velocity, and water-surface measurements are the most troublesome.

Ordinarily the discharge passing a structure can be measured at a gaging station downstream. During flood flows, however, it is frequently impossible to obtain an accurate measurement because of a change in river section due to bed movement, or because the peak flow may pass before a complete measurement is recorded. Occasionally a flood will occur at night, whereupon an automatic gage, if installed, must be relied upon or the measurement is lost. On the other hand, it is sometimes necessary to wait many years before a structure operates, and even then a sufficient range of head and discharge may not be obtained. Frequently the discharge passing a structure may be derived from the simultaneous operation of a spillway, outlet works, and power house. In such an event it is usually necessary to wait until the flow is derived from only one source, but by that time the range of measurements desired may have passed. Similarly, if a confluence of streams exists between the structure and gaging station, or if heavy rainfall occurs, a discharge measurement adjusted to be applicable to the structure may be of questionable value.

Perhaps the most trouble is encountered when it is desired to obtain velocity and water-surface measurements, not only for comparison with the model, but also for procuring fundamental data. If the flow travels at slow velocities and is confined in narrow channels, these measurements may be made with a minimum of difficulty. If the flow travels at high velocities, however, the measurements are extremely troublesome, regardless of the size of channel. For example, how would one measure accurately the velocity and water surface of the flow in the spillway section of a dam in a western state or of the dam in question here where the velocity is in excess of 100 ft per sec, and the water surface is affected by air entrainment? It is impracticable to use current meters or pitot tubes to measure the velocity, or point gages to obtain the water surface.

The futility of using pitot tubes was realized at the dam discussed here, more from the trouble of maintaining the equipment than from the magnitude of the quantities to be measured. During construction three streamlined, bronze piers, about 21 in. high, were fastened to bronze base plates at the entrance and on the lip of the spillway bucket. Each bronze pier supports five stainless-steel pitot tubes whose static and dynamic openings are connected within the piers to hydrostatic pressure cells similar to those used for measuring seepage pressures in earth dams. Each cell is connected to an instrument panel in a gallery at a higher elevation by copper tubing extending through the dam. To provide check measurements, a by-pass is formed at each cell and connected by copper tubing to manometers in the gallery. In spite of all the precautions made, many of the copper tubes leading to the gallery became plugged and all attempts to clear them failed. Some of the diaphragms of the pressure cells were ruptured during the attempts to clear them. It is also feared that debris in the tailwater has plugged or coated some of the pitot-tube openings and may have damaged the entire assembly from impact, which may be even more severe since the start of spillway operation in June, 1942. As a result, it is doubtful that any reliable measurements can be taken with this equipment, even though carefully planned and all precautions taken to assure an efficient arrangement. It was realized that the pitot-tube assembly might be inadequate, so the salt-injection method will be attempted making use of electrode plates, which were also installed at intervals down the spillway section of the dam. As yet no suitable methods have been developed to measure the water surface for the high-velocity flow occurring in spillways at high dams or in other open channels.

Difficulties are also encountered when measuring velocities in large outlets under high heads because of the tremendous forces involved, which either necessitate extremely rigid instruments or prohibit the use of instruments entirely. Lack of access also eliminates any measurements, particularly if previous preparations have not been made for installing the necessary equipment during construction.

The success with which other measurements can be made in the field, such as pressures on boundary surfaces, vibrations, and air-demand, depends more on the careful planning of the instruments, the ability to foresee their workability, and their installation, than upon the difficulties of actually being able to obtain the measurements.

Planning and Cost.—Since the success of any verification depends upon the flow measurements made on a model and its prototype, it is compulsory to formulate plans for the comparison during the laboratory experiments and during the design of the structure. Such plans should provide for quantitative model data covering several flows, not just the maximum, and for the installation of piezometers, gages, and special instruments in the prototype design as dictated by the model tests. As a result of careful planning, the unfortunate condition of having prototype data in one range and model data in another will be avoided, even though it might be possible to regulate the prototype tests to conform to similar ones of the model—a condition which rarely exists.

The success of any planning, moreover, depends upon the circumstances under which the laboratory tests are made. Usually tests are performed under considerable strain because of the limited time available after the preliminary design is submitted for testing and before the final design is prepared for construction. Discovery of faults in design during construction also requires model experiments to be made quickly so as not to interfere with the progress of construction. Because of this constant rush, the model data usually do not cover the desired range, thorough checks on the data are not possible, and the adequacy of the prototype measuring equipment may suffer.

If some flow measurements have been made over a period of years at a structure never previously tested by a model, it is of considerable interest to construct a model and secure the data necessary for a comparison. This reversed procedure has been tried at the University of Iowa, at Iowa City, Iowa (see paper by Mr. Soucek in this Symposium), as an academic study, and was used recently at the Bureau of Reclamation, but for the primary purpose of revising hydraulic conditions at an existing structure.

The cost of taking field measurements will be greatly reduced if plans provide for the installation of equipment during construction. Once the proper arrangements have been made, the actual cost of making the measurements will be small in comparison with the entire cost of the structure itself. The cost of instrumentation, on the other hand, is relatively high and in many cases prohibits any field measurements.

Properties of the Model.—Not all of the difficulties in making comparisons are attributed to the prototype tests; in fact, most of them must be traced directly to the model experiments, assuming, of course, that the prototype data are reasonably accurate. This is evident since any comparison actually is a check on the model tests, not on the prototype. In open-channel studies a small error is introduced because it is usually impossible to select a scale ratio that will permit rigorous similitude between model and prototype roughness, the model surfaces being too rough. During experiments on different scale models of sluices in the Assuan Dam (4), in Egypt, it was found that models having a surface roughness commonly used today agreed with the prototype discharge within 2%. When the model surfaces were roughened artificially with sand, however, the model discharge decreased 5%. From measurements on five geometrically similar models of spillways, F. Eisner (5) showed that there was a definite variation in the coefficient of discharge with the scale ratio. The relative roughness of the boundary surfaces was believed to be the important factor, the effects of viscosity being almost negligible. Because of the uniform variation between the coefficients of discharge and the relative roughness, Eisner predicted the prototype coefficients reasonably well, in comparison to extrapolating by the Froude number.

A much greater error is introduced in model studies of closed conduits because it is practically always impossible to operate the model at Reynolds numbers as high as those obtained at the prototype. It is important to realize, therefore, that the model friction losses are too large, and that a correction should be made to the model data before estimating any corresponding prototype values. Other errors introduced by such fluid forces as capillarity and

elasticity are negligible in the problems usually studied in hydraulic laboratories. In any event it is well to know the requirements for similitude, how to apply them, and their effects on the results.

In regard to the model data, they are usually more accurate than the prototype measurements, since errors in the field are not controlled as easily as they are in the laboratory. In this regard it should be determined if the model and prototype dimensions are geometrically similar, because frequently the field structure may not be constructed in accordance with the dimensions used for the model.

Prototype Flow Measurements

Since it is necessary to take nearly the same measurements in the field as in the laboratory, instruments should be available for measuring the discharge, pressure on boundary surfaces, water-surface elevations and profiles, velocity, vibration, air-demand, bed movement, and for photography. Keeping in mind the difficulties mentioned in obtaining prototype measurements, several well-known methods will be outlined. Although the instruments used in the laboratory and in the field are basically similar, the prototype equipment must be designed to withstand tremendous forces and be convenient for use. With little precedent to follow, one must rely on good judgment for the design of the instruments and, at the same time, be able to foresee their efficacy.

Discharge.—Open-channel flows can usually be measured by current meters, pitot-tube or pitometer traverses, weirs, Parshall flumes, floats, and the salt-injection method. The particular method used depends on the accuracy required, the size and type of channel, quantity of water, and its velocity. Accordingly, a current meter is used most frequently in rivers, lined channels, and in canals. Smaller quantities in canals are measured with weirs or Parshall flumes. For high-velocity flow in lined channels, the salt-injection method is used unless the discharge measurement can be made at some other place where a more convenient method could be adapted.

Discharges in closed conduits are obtained by pitot-tube traverses, the salt-velocity method, the Gibson method, and by venturi meters when installed in the conduit. These methods are adaptable only if plans have been made for their use, since it has been pointed out that it is often impossible to gain access to many conduits in dams. If access is impossible, the discharge probably could be obtained downstream at a gaging station.

In most of these methods a calibration of the instruments is required, and it is debatable whether the calibration of some of them will be the same at the test site as it is at the place of calibration. Using a rating curve at the prototype which has been established from model tests obviously defeats the purpose of making a comparison.

Pressure.—Piezometers, which are used almost exclusively to obtain pressures on boundary surfaces, should be installed on the prototype in positions similar to those on the model. The installation of piezometers in two of the 102-in. river outlets of the dam is offered as an example. Piezometers in these outlets have been placed on the vertical and horizontal diameters. Each opening consists of a high-tension bronze plug drilled with a $\frac{1}{4}$-in.

hole and screwed into the steel plate lining of the conduits, with the orifice end projecting about $\frac{1}{16}$ in. past the interior surface. After this projection had been ground flush with the surface lining, a $\frac{1}{16}$-in. radius was formed in the orifice end and all burrs were removed. A short length of $\frac{1}{8}$-in. standard brass pipe connects each plug to a steel junction box, where a reducing coupling is used to connect each piezometer plug to $\frac{3}{8}$-in. outside diameter by 0.035-in. wall, soft copper tubing. Junction boxes placed opposite each plug are connected by 2-in. galvanized conduit, through which the soft copper tubing is collected, and lead to six headers recessed in an inspection gallery below the outlets. The headers, one placed above the other, are made of 2-in., extra-strong brass pipe, with $\frac{1}{8}$-in. brass air cocks placed in rows along the top of each header to make connection with each piezometer line. Each header is connected to a high-pressure line at one end and to a bleeder pipe at the other end, which, in turn, is connected to a pressure measuring instrument through a by-pass.

A similar method could be used for measuring pressures on concrete surfaces, except the piezometers would be attached to a metal plate set in the concrete. In any event the installation must be made during construction.

To measure the pressures, it is common practice to use water or mercury manometers, and dial gages. Manometers are not always satisfactory because of their fragility, the fluctuations of the mercury columns, and the magnitude of the pressures frequently encountered. Commercial-type dial gages are usually not precise enough. A fluid-pressure scale of the type used for turbine testing proved satisfactory in high-head tests at the two dams. Pressures as great as 300 lb per sq in. can be recorded within 0.25 lb per sq in. to pressures of 100 lb, and within 0.25% at any pressure greater than 100 lb with this instrument, which is compact and of simple construction. When reading piezometer pressures, the pressure is transmitted to a copper tube grid mounted in gimbals in a weighing beam in the base of the instrument. The weighing piston, which is lapped fit in the cylinder, is restrained from being forced upward. The resulting downward reaction on the weighing cylinder is transmitted, therefore, to the lower weighing beam, and measured on an upper scale beam mounted above the base. The weighing piston is rotated by a small electric motor while readings are being made to avoid static friction with the walls of the cylinder.

Pressure cells used in combination with a group of piezometers and an oscillograph may be used, but the cost of such equipment is prohibitive in many cases unless it is available from previous experiments. This method has one advantage in that a continuous record of the pressure fluctuations is recorded on the film or sensitized paper in the oscillograph.

In making any pressure measurements either on the model or prototype, the care with which the piezometer openings are installed cannot be overemphasized. Any burr will affect the reading seriously, and excessive surface roughness either upstream or downstream from the piezometer opening will influence the measurements. It is essential to have a high-pressure water line for flushing the piezometer lines clear of air bubbles, and the tubing from

the piezometer to the measuring instrument should be alined, if possible, to eliminate any bends or sags that might trap air.

Water Surface.—Although these measurements are not as important as the discharge and pressure, nevertheless a check is desirable on such design assumptions as freeboard and roughness coefficients. In connection with the work of the Society's Special Committee on Hydraulic Research, the problem of air entrainment in high-velocity flow requires, in part, a measurement of the water surface in open channels at field structures. In comparison to the low-velocity flow in models where the water surfaces are relatively smooth and well-defined, there is actually no definite water surface for the high-velocity flow at field structures, but only a mass of spray and aerated water. Efforts to point-gage such a water surface in a narrow wasteway in the Yakima project, Washington, were successful in a measure, but only an "average" water surface was actually obtained.

Although point gages may be useful for measuring water surfaces in narrow channels, it might be more feasible at wide channels to use a system of cables to manipulate a heavy target, which could be located by transits. A profile of the water surface could also be obtained by suspending a plumb bob from the training walls, or from a grid painted on the training walls; but such measurements would be affected by the wall friction.

For measuring water-surface elevations in a reservoir, use has been made of the familiar staff gage and of several commercial instruments. Those based on the principle of a long air column transmitting the pressure should be avoided because of the effect of temperature change on the air column, and because of possible air leaks in the tubing.

Velocity.—The methods available are directly associated with discharge measurements, so use is made of current meters, pitot tubes, salt-injection method, and floats. Although somewhat unsuccessful at Grand Coulee Dam, the installation of pitot tubes, as discussed herein, may prove successful under different circumstances. The accuracy required for model-prototype comparisons eliminates the use of surface and subsurface floats.

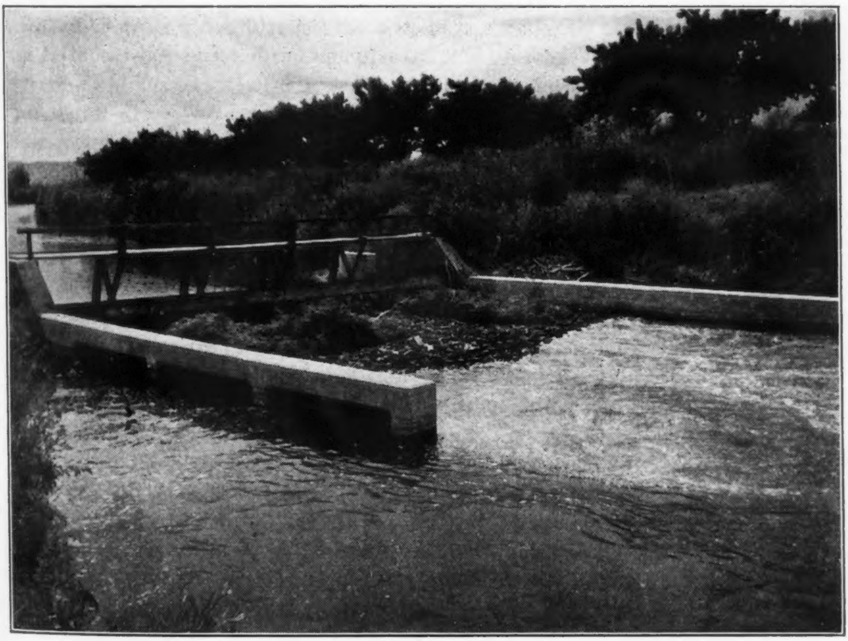
Vibration.—It is frequently desirable to measure the vibrations occurring at a prototype structure, not necessarily to compare with the model to establish a relation between the two, but more to ascertain what effect the vibrations may have on the structure itself. The flutter of nappes discharging over the lips of drum gates on the crest of movable weirs or dams is one example of such vibration (6). The discharge at large valves and regulating gates also is frequently accompanied by vibrations. Accelerometers and pressure cells were used with an oscillograph to measure vibrations due to fluttering nappes at a dam in another western state. The instruments were designed on the principle that a change of resistance occurs with a change in pressure on a pile of carbon disks. The accelerometers measured the accelerations in three directions in the dam, and the pressure cells were used to measure the amplitude and frequency of the inducing forces exerted on the dam by the flutter of the nappes spilling over the lip of drum gates. The instruments were calibrated by noting the deflections produced on the moving elements of the oscillograph under

(*a*) 1 : 15 Scale Model, Before Revision

(*b*) Prototype Before Revision

FIG. 1.—CHECK DROP No. 4

(c) 1 : 15 Scale Model, After Revision



(d) Prototype After Revision

(DISCHARGE, 1,300 CU FT PER SEC)

various conditions created on a shaking table. Less complicated instruments, such as vibrating reeds, which give only the frequency, have been used elsewhere, the amplitude being determined by magnifying the vibration and using an oscillograph.

Air-Demand.—It is often necessary and desirable to measure the quantity of air required on a prototype to relieve sub-atmospheric pressures under nappes, in conduits downstream from needle valves and regulating gates, and in regulating gates and valves themselves. It is a difficult problem to determine the size of air ducts required on the prototype by model tests, because of the uncertainty of the similitude involved; thus any check on the prototype will aid considerably in establishing the transference relations. To measure air quantities on the prototype (and in the model) an anemometer, orifice plate, or pitot tube could be installed in the air ducts. Although not entirely suitable for prototype measurements because of the large quantities involved, a rotameter may be used for model tests.

Bed Movement.—The scour to river channels downstream from dams and spillways is obtained by surveys or by soundings. These measurements, however, can be used only for a qualitative comparison.

Photography.—Still and motion pictures of all flow conditions observed both in laboratory and field work are invaluable for purposes of comparison. Frequently a startling similarity is seen between a model and prototype photograph of some flow phenomenon. For still pictures, the photographic equipment should include at least a standard view camera and a miniature camera, both with fast lens. The motion-picture camera should be equipped for slow-motion. The higher the number of frames per second, the better, because many prototype flows cannot be studied unless considerably slowed down in the motion pictures. Telephoto lens attachments are considered essential for both the still and motion-picture cameras.

Examples of Qualitative Comparisons

Irrigation Structures.—Such relatively small irrigation structures as diversion dams, check drops, wasteways, and turnouts offer opportunities for making model-prototype comparisons because of their continuous operation at various discharges during an irrigation season. The primary concern at these structures is to prevent scouring of the canal downstream by using a hydraulic-jump stilling basin wherever possible. Consequently, it is sufficient to make only a qualitative comparison of any erosion that occurs. Most of the small irrigation structures in use today are based on designs developed over a period of years, so that model studies have not been utilized except for revising some existing structure of inadequate design. Such a condition required a laboratory investigation of Check Drop 4 in the Main Canal of a western project.

The Prototype.—Because of a gradual growth in irrigation, the Main Canal had to be increased in capacity. To maintain a proper gradient in the canal, twenty-three check drops were installed at intervals of about one and one-half miles. These drop structures, which were built during the period 1907–1916, were similar in design, but decreasing in size downstream as the

discharge in the canal became less due to deliveries. Each structure consists of a rectangular concrete basin divided into bays by concrete piers which are surmounted by steel brackets. By placing flashboards between each pair of brackets, the depth of flow between successive drops can be regulated. Wing walls at each end of the concrete basin extend into the canal banks at right angles. The drop in water surface at these structures varies from 30 in. for the maximum to $7\frac{1}{4}$ in. for the minimum, with a maximum discharge of 1,300 cu ft per sec at the first drop gradually decreasing to 520 cu ft per sec at the last one.

Not many irrigation seasons had passed before excessive scouring developed in the canal immediately downstream from the drops, particularly at the larger ones. All attempts to check the erosion failed, until finally the width of the canal below the drops was increased as much as 50% in some cases. In 1938 the problem of eliminating this scour was assigned to the hydraulic laboratory of the Bureau of Reclamation, Denver, Colo.

Model Tests.—Since the drops were similar in design, Check Drop 4 was selected for study, and a 1 : 15 scale model was constructed. The initial tests in the laboratory demonstrated that the model would reproduce the flow conditions and scour existing in the field (Figs. 1(*a*) and 1(*b*)). Further analysis disclosed that, because of the small drop in water surface, standing waves developed instead of a more efficient energy dissipator, such as the hydraulic jump. As a result, high surface velocities proceeded downstream in the center of the canal, thereby producing large return eddies along the canal banks which eroded them severely.

A satisfactory solution to this problem was not readily apparent, for not until many designs had been tested was a solution found which would eliminate the unfavorable conditions completely. This solution consisted of adding curved training walls in the entrance to the drop, extending vertical walls and a concrete floor downstream to form a stilling pool, and placing a deflector between these walls directly above the concrete basin to force the flow under the tailwater. These revisions created a more uniform flow distribution through the structure, and provided energy dissipation sufficient to eliminate the high surface velocities which had caused side eddies to scour the canal banks.

Comparison.—After the prototype had been revised, observations of its performance showed that it was acting in a manner similar to that predicted by the model tests. Figs. 1(*c*) and 1(*d*) show the revised design of the model and prototype. The collection of debris at the prototype structure is evidence of roller action in the stilling pool. This is indicative of energy dissipation throughout the entire depth, instead of only on the surface as noticed in Figs. 1(*a*) and 1(*b*).

During the laboratory studies a similar collection of debris was noticed, but it was given little consideration since it interfered with photographing the action of the model. In the field, however, it has been of considerable value in clearing the canal of large quantities of debris, conveniently removed at the drop structure. Three miles downstream at Drop 6 (which was also revised), there is little trash remaining in the canal.

The results of this comparison are of considerable satisfaction to those familiar with the problem, but as previously mentioned such a correlation is more convincing to those directly concerned. Nevertheless, this study is offered as convincing proof that model tests are an excellent tool for the hydraulic engineer, and that they can be relied upon to a high degree.

Examples of Quantitative Comparisons

The Prototype.—Sixty 102-in. outlets were provided in three tiers of 20 each at the dam to release a total discharge of 225,000 cu ft per sec (Fig. 2). Each outlet has a bellmouth entrance followed by a length of straight

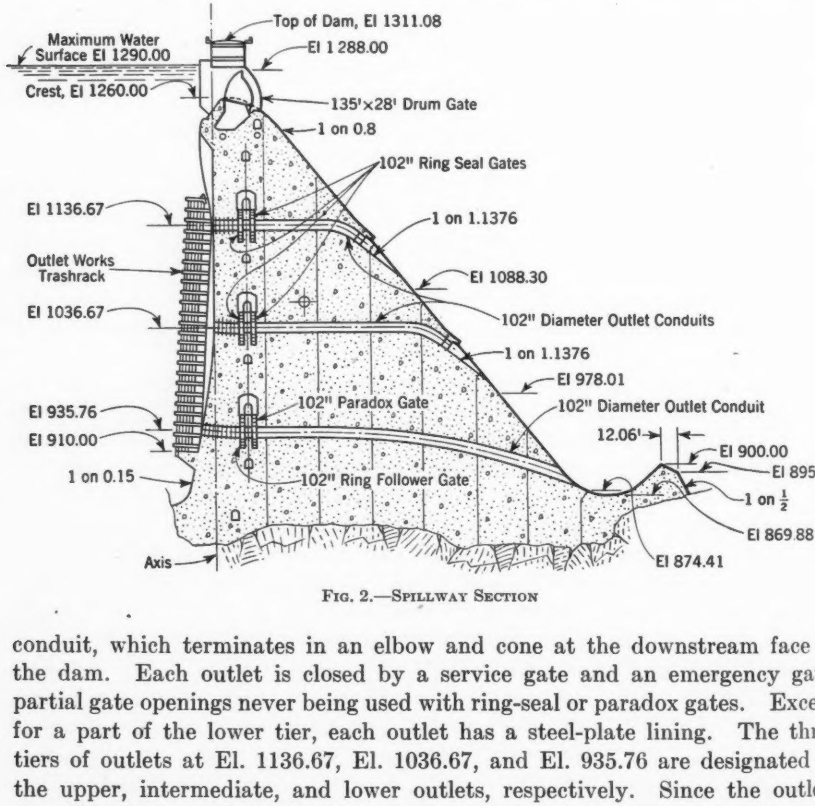


Fig. 2.—Spillway Section

conduit, which terminates in an elbow and cone at the downstream face of the dam. Each outlet is closed by a service gate and an emergency gate, partial gate openings never being used with ring-seal or paradox gates. Except for a part of the lower tier, each outlet has a steel-plate lining. The three tiers of outlets at El. 1136.67, El. 1036.67, and El. 935.76 are designated as the upper, intermediate, and lower outlets, respectively. Since the outlets occur in pairs in each tier, 15 ft on centers, further designation is given by east (E) and west (W), and by the block number in which they occur. Fig. 3, for example, shows upper outlet 63W discharging down the face of the dam.

During the design of the outlets, plans were formulated for installing piezometers in one intermediate outlet, 51E, and one upper outlet, 51E, 100 ft immediately above. Piezometers were not placed in the lower outlets because they were not completely steel-lined and because detailed model studies were

not made of them. In locations similar to those in the model, a series of eleven piezometers have been placed in the bellmouth entrances along both the invert and crown (vertical center line), and on the sides (horizontal center line). Similarly, the horizontal part of the two conduits contains twelve piezometer rings in the intermediate outlet, and ten in the upper outlet; both have ten rings in the elbow and cone. The installation methods have been described and the vertical section of an intermediate outlet shown in Fig. 4 indicates the distribution of piezometers.

Model Tests.—A résumé of the 1 : 17 scale model tests on the outlets of the dam has been given in another paper (7). The tests were concerned with the bellmouth entrance, the profile of the outlets, the elbow and cone, and the deflectors placed in the spillway immediately above the exit of the upper and intermediate outlets, which can be seen in Figs. 3 and 4(*a*). Measurements on the model were concerned primarily with the pressures on the boundary surfaces, measured with water and mercury manometers, and the discharge for several heads, obtained from weir measurements.

Only the model data from the intermediate outlet tests will be used for comparing with the prototype. Some tests were made in the laboratory on the lower outlets, but no piezometers were installed in the prototype, and since the upper outlets are quite similar to those in the intermediate tier, additional tests were not required.

Prototype Tests.—Twenty-three tests were made on the intermediate and eight on the upper outlets. Heads above the exit ranged from 28 ft to 234 ft

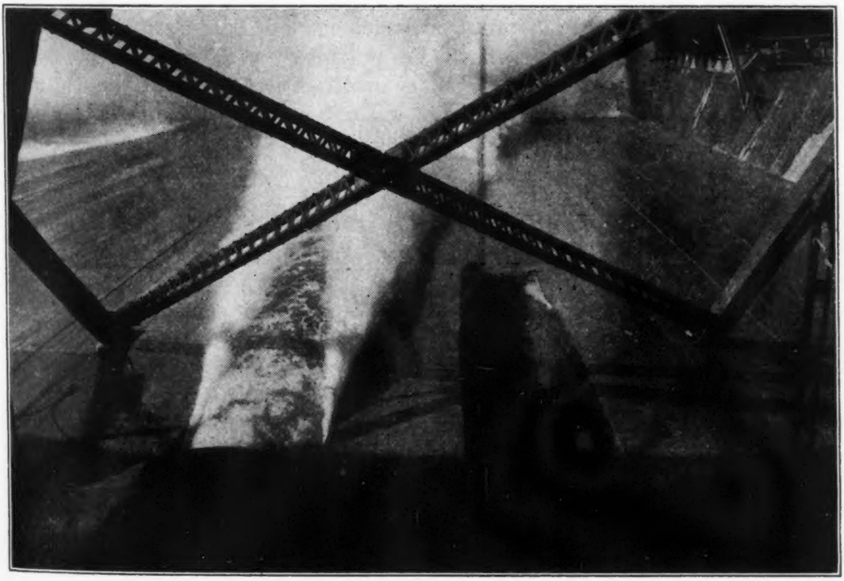


FIG. 3.—UPPER OUTLET 63W; DISCHARGE 2,400 CU FT PER SEC

for the intermediate outlet, and from 33 ft to 133 ft for the upper outlet. In addition to measuring pressures with the fluid pressure scale, the reservoir elevation, water temperature, tailwater elevation, discharge, and the number of outlets open in each tier were recorded. To make comparisons with the

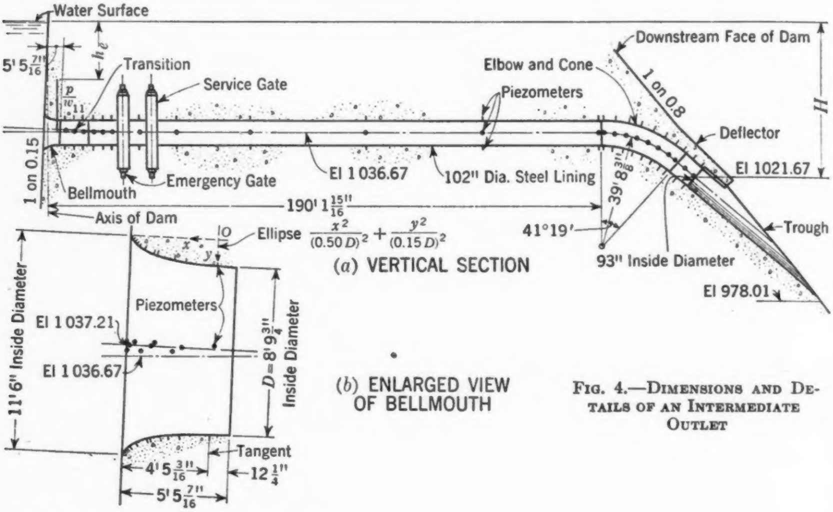


Fig. 4.—Dimensions and Details of an Intermediate Outlet

model of the intermediate outlet, it was possible to use only those tests in which the adjacent outlet, 51W, and the upper outlet, 51E, were closed. This not only conformed with the laboratory arrangement, but it also eliminated from consideration any secondary effects on the pressures. These effects were discovered first during a separate model investigation of the bellmouth, and later during the prototype tests when it was noticed that pressures in the bellmouth were slightly lowered when the adjacent outlet 51W was operating, while pressures in the elbow and cone were raised when the upper outlet 51E was operating.

Notation.—The letter symbols used in this paper are defined where they first appear and are assembled for convenience of reference under "Bibliography and Notation" at the end of the Symposium.

Model-Prototype Pressure Characteristics.—The first comparison given in Fig. 5 is intended to show the characteristics of the hydraulic gradient. Because the field tests could not always be made when the reservoir reached a corresponding elevation, the model and prototype heads shown are not comparable. Upon first consideration it would seem desirable for the heads to agree, but even if they did, an agreement between the model and prototype pressures would be lacking. The reason for this is evident from Fig. 6, where it may be seen that the model tests were made at much lower Reynolds numbers (**R**); hence the friction losses

$$f = \frac{h_f}{\frac{L}{D}\frac{V^2}{2\,g}} \qquad (1)$$

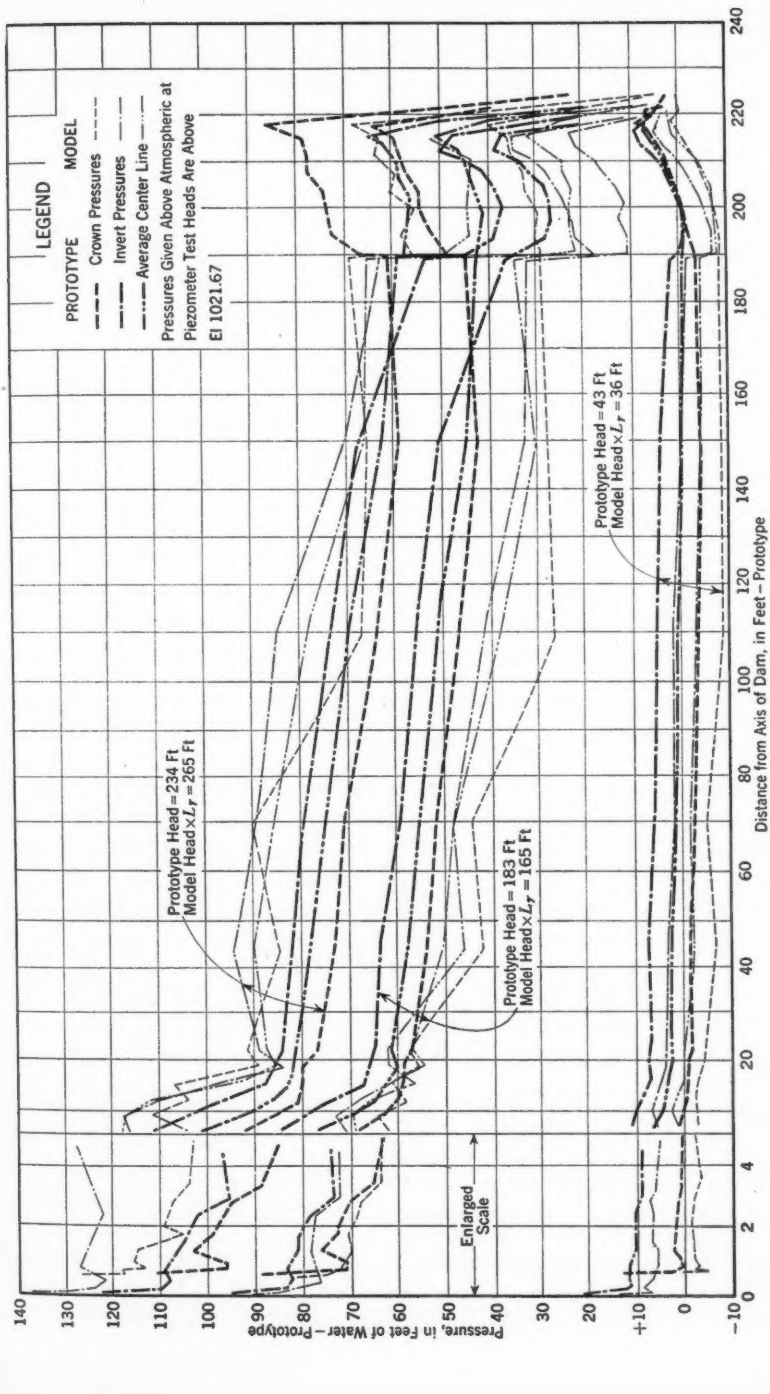


FIG. 5.—PRESSURE CHARACTERISTICS, MODEL AND PROTOTYPE

were proportionately greater (f = coefficient of friction; h_f = loss of head due to friction, feet of water; V = average velocity through conduit; L = length of section, in feet; D = diameter of conduit; and g = acceleration due to gravity). Consequently, when the model pressure heads are scaled to the prototype by the Froude law, they will be lower than corresponding proto-

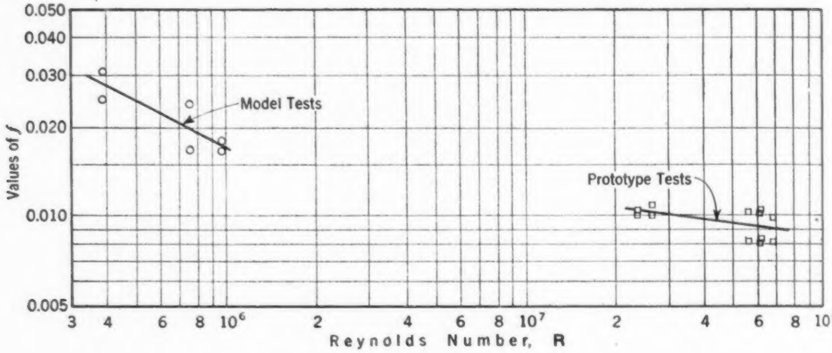


FIG. 6.—COEFFICIENTS OF FRICTION

type values. This fact is generally well-known and is only mentioned to show the effect of not satisfying the similitude requirement that the model should be tested at the same Reynolds number as the prototype. This condition is incompatible with hydraulic models based on Froude's law, in addition to the fact that the extremely high velocities required to comply with the Reynolds law are impossible to obtain in many laboratories. In any event the model results are on the side of safety.

Notwithstanding the lack of complete similitude, a study of Fig. 5 indicates a certain similarity between the model and prototype gradients. Whereas the prototype data are quite consistent, the model data appear to be erratic at some of the piezometers.

Pressure Drop in Bellmouth.—To make a more valid comparison than the one in Fig. 5, the pressure drop in the bellmouth has been analyzed. By choosing a short length of conduit, the effect of the differences in friction is negligible, and by using dimensionless parameters, comparable heads are not required. Accordingly, as shown in Figs. 7 and 8, curves were drawn for the crown and invert using as ordinates $\frac{H - h_x}{H - h_{11}}$, and as abscissas $\frac{L_x}{L_{11}}$, in which H is the head above El. 1036.67; h_x is the pressure at any piezometer in the bellmouth referred to El. 1036.67 as a datum; h_{11} is the pressure at the reference piezometer, number 11, referred to the same datum; L_x is the distance from the face of the bellmouth to any piezometer; and L_{11} is the distance from the face of the bellmouth to piezometer number 11, as shown in Fig. 9. Then values of $\frac{H - h_x}{H - h_{11}}$ express the pressure drop at any piezometer as a percentage of the total, whereas the distance ratio $\frac{L_x}{L_{11}}$ eliminates any small differences

between the location of piezometers in the model or prototype. Two of the prototype tests were omitted from the diagrams so as not to confuse them with more superimposition of the points, but all of the model values are included. The actual pressures at the piezometers in the bellmouth were all positive (above atmospheric) except for low heads, and these did not exceed a vacuum of 10 ft of water.

The prototype curves illustrate that the percentage pressure drop is practically constant at nearly all the piezometers regardless of the head. This is not true for the model tests, even though the average values of the model are within approximately 5% of the prototype. Reasons for the dispersion of the model data may be explained by experimental error, different approach conditions, and unstable flow in the model at lower Reynolds numbers, although above critical Reynolds numbers.

Experimental errors are probably due to the installation of the model piezometers, even though great care was taken to install them properly. Nevertheless, any minute obstructions on the boundary surfaces and in the piezometer openings have serious effects. The laboratory experiments, moreover, were made under considerable duress to enable the design to keep ahead of construction in the field. Consequently, the customary close checks on the data could not be made as complete as they would have been under a more normal procedure. The different approach conditions refer to the absence in the laboratory of a trashrack upstream from the bellmouth. However, it was observed from separate tests on the bellmouth that the trashrack effects were negligible.

Evidently the differences in friction play a small part even though a comparison is confined to the bellmouth alone. No estimate can be made readily of the relative roughness of either the model or prototype surfaces, but since the type of surface in each is nearly the same, the relative roughness of the model would be greater. Consequently at lower Reynolds numbers, the resistance factor for the model will vary considerably more within its range of tests. It can be assumed further that the model tests were for rough flow, whereas the prototype tests were more nearly smooth flow. Finally, it might be added that a transition from a laminar sublayer to a turbulent sublayer will develop less rapidly in the model, thus influencing the velocity distribution and pressures. It is reasonable to assume, therefore, that the model flow is not sufficiently stable to give the consistent data obtained for the prototype. Accordingly, it is believed that the laboratory tests should have been made at greater velocities (higher Reynolds numbers), disregarding heads based on the scale ratio of the model. Had this been done, the model data as analyzed in Figs. 7 and 8 would have been more consistent and would have approached the prototype values more closely, although 5% is certainly good agreement for the average curves.

Pressure Drop in Elbow and Cone.—Dimensionless curves have been drawn in Figs. 10 and 11 to show the pressure drop in the elbow and cone in a manner similar to that described for the bellmouth comparison. To eliminate the differences in friction up to the elbow, the pressure drop has been expressed as

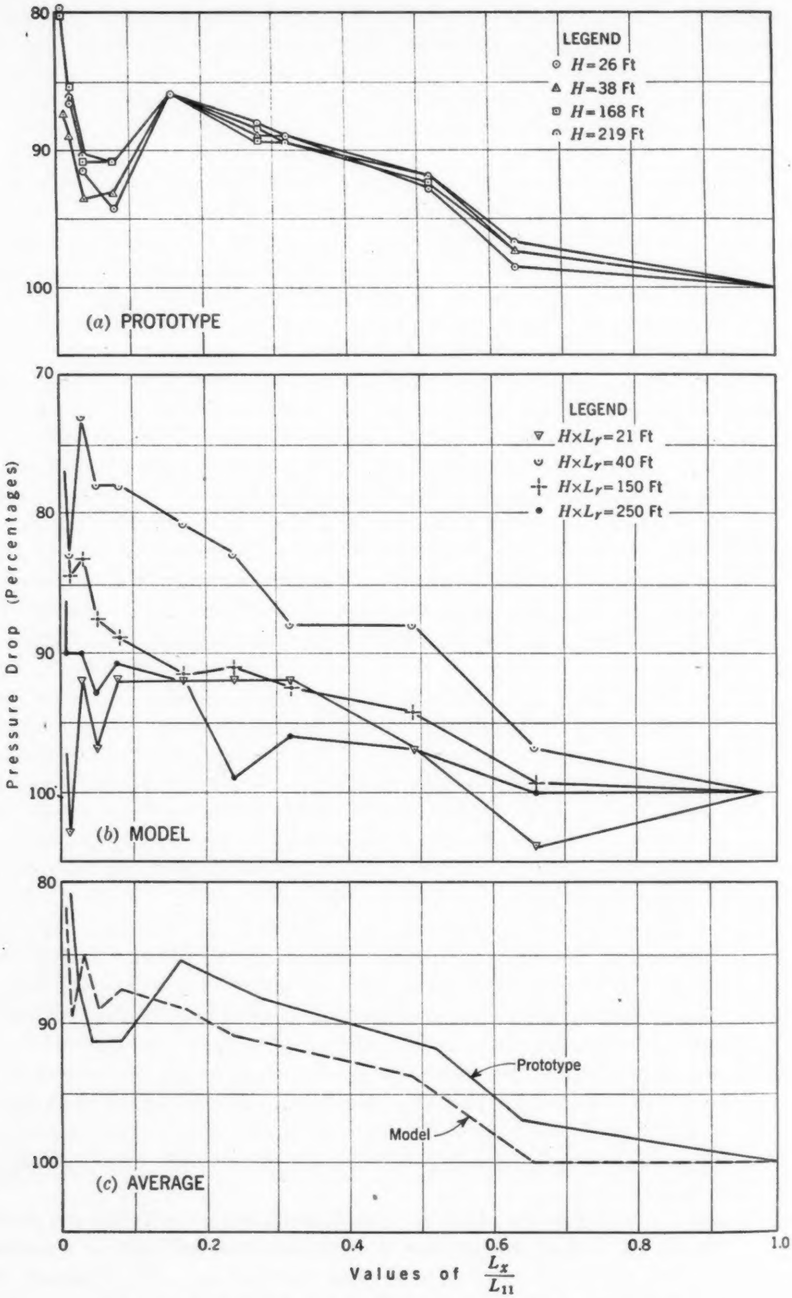


FIG. 7.—COMPARISON OF PRESSURE DROPS ON CROWN OF BELLMOUTH

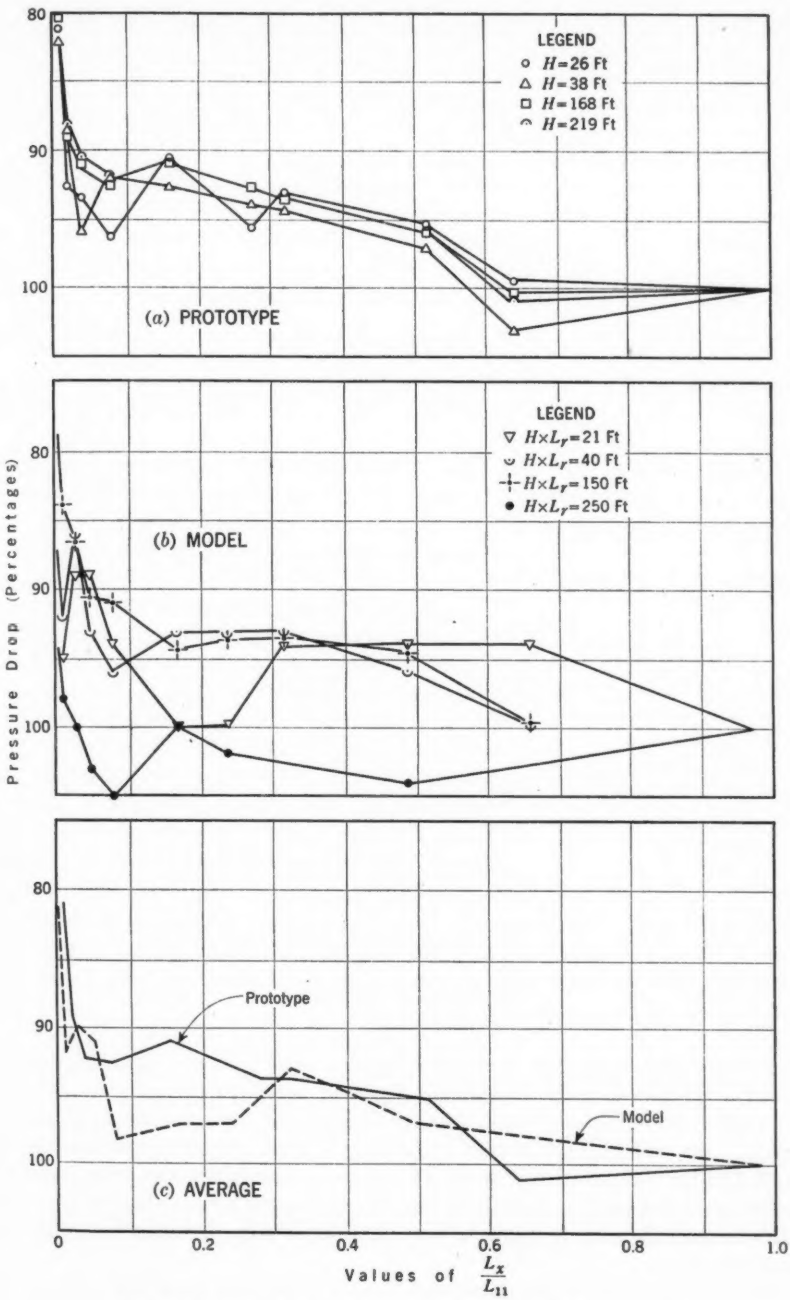


FIG. 8.—COMPARISON OF PRESSURE DROPS ON INVERT OF BELLMOUTH

$\frac{h_{23} - h_x}{h_{23} - h_{33}}$, in which h_{23} and h_{33} are the pressures referred to El. 1021.67 as a datum, at the start of the elbow and at the end of the cone, respectively; h_x is the pressure at any piezometer referred to the same datum; L_x is the distance along the center line from piezometer 23 to any piezometer; and L is the center-line distance from piezometer 23 to piezometer 33 (see Fig. 12). In Figs. 7 and 8, two prototype tests have been omitted for clarity, but all model tests are shown. The heads, however, are referred to El. 1021.67 instead of El. 1036.67, and average center-line pressure drops have been added. The reason for negative ordinate values for the pressure drop on the crown is due to an increase in pressure from piezometer 23, thus making the numerator negative in the ratio $\frac{h_{23} - h_x}{h_{23} - h_{33}}$. Although not shown in Figs. 10 and 11, the center-line pressure drop falls as a good average between the curves for the crown and invert for both model and prototype. This may be seen from the curves given in Fig. 5.

Figs. 10 and 11 show the data for both the model and prototype as almost equally consistent, the model values still being slightly erratic. Reasonable agreement, however, is lacking. In general, the pressure drop indicated from the model tests is about 30% greater than is shown from the field measurements. This unsatisfactory agreement may be explained by the principal reasons applied to the bellmouth comparison; namely, experimental errors and unstable flow for the range of Reynolds numbers in the model.

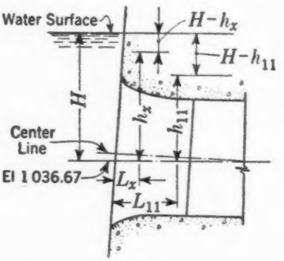


Fig. 9

In computing the model data for the elbow and cone, it was evident from the excessive dispersion of the points that some of the pressure readings were in error. To adjust these, a curve of head versus pressure was plotted for each piezometer. Since these curves should be a straight line it was possible to detect which readings were erroneous and to adjust them to the line. Unfortunately, if the pressures at the start of the elbow, h_{23}, and at the end of the cone, h_{33}, are not initially correct or are not properly adjusted, then the total pressure drop $h_{23} - h_{33}$ causes an error in the ratio $\frac{h_{23} - h_x}{h_{23} - h_{33}}$ plotted in Figs. 10 and 11.

One of the field tests was similarly adjusted where obvious errors were noted. By checking the piezometer numbers attached to each header, it was found that the erroneous readings were all in a group located in one piezometer header. It was concluded that the pressure from another piezometer in the same header was being registered at the time this group was being recorded. An error of this nature is easily made when so many piezometers are required, unless particular care is taken to be certain that only one piezometer is being read at a time.

Since the length of the elbow and cone is nearly six times that of the bellmouth, it is evident that the differences between the model and prototype

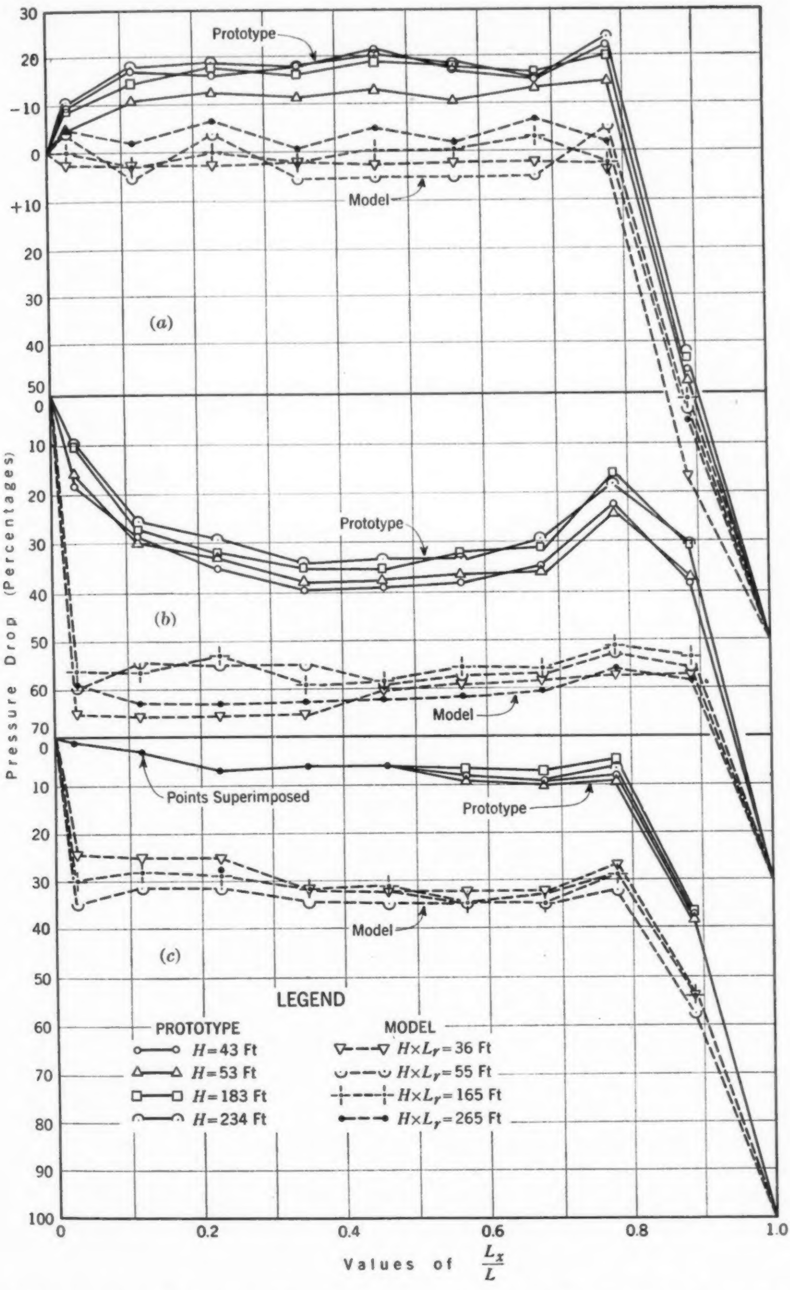


FIG. 10.—COMPARISON OF PRESSURE DROPS IN ELBOW: (a) CROWN; (b) INVERT; AND (c) CENTER LINE

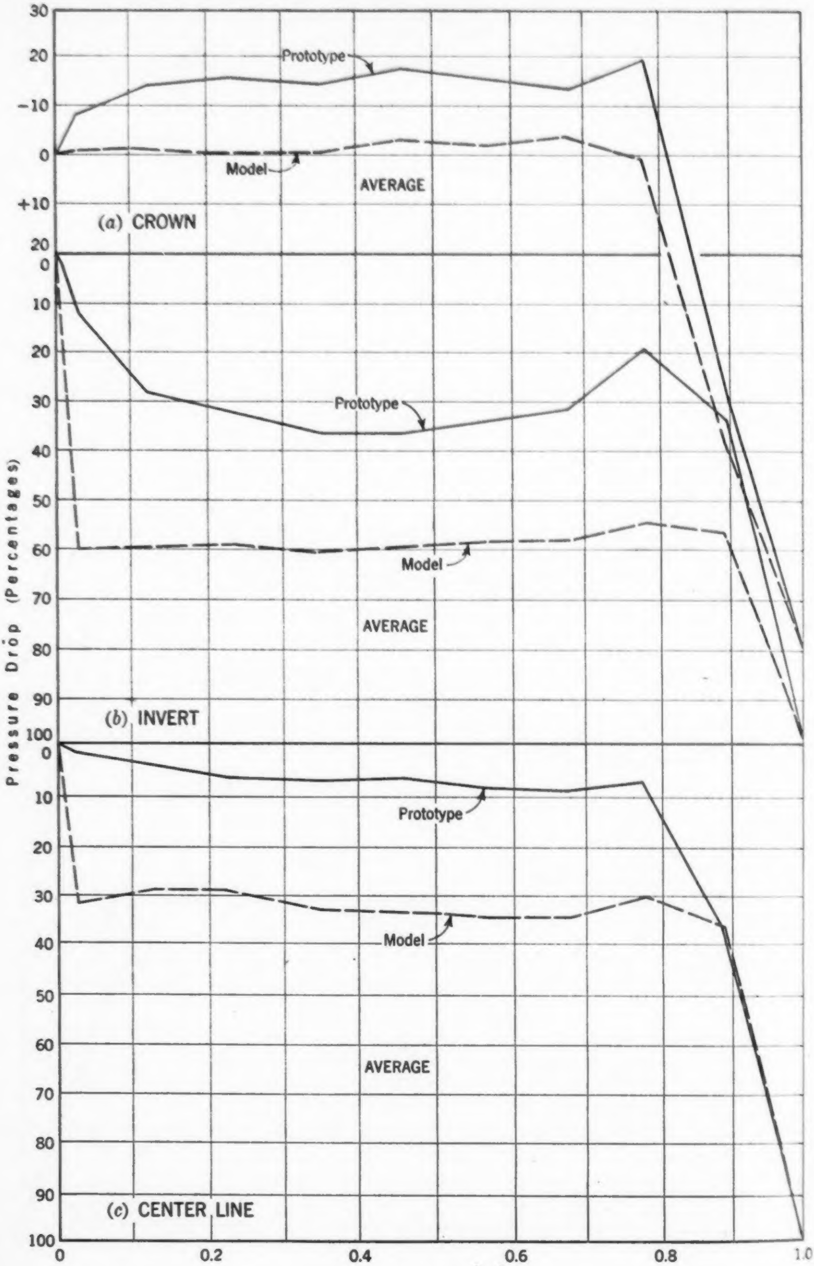


FIG. 11.—COMPARISON OF PRESSURE DROPS IN CONE

friction will have still more effect on the comparison, even though a study of the losses shows the bend loss or centrifugal effect to be considerably greater. The latter seems to be noticeable in the prototype curves in Figs. 10 and 11, where the points are more scattered than for the bellmouth measurements shown in Fig. 9. From tests on other structures, it is known that a slight change of diameter of the cone exit will cause appreciable changes in the pressure gradients through the penstock, making comparison more difficult. Time did not permit a more complete examination of the data for the elbow and cone, which might disclose a more valid method for comparing the pressure drop.

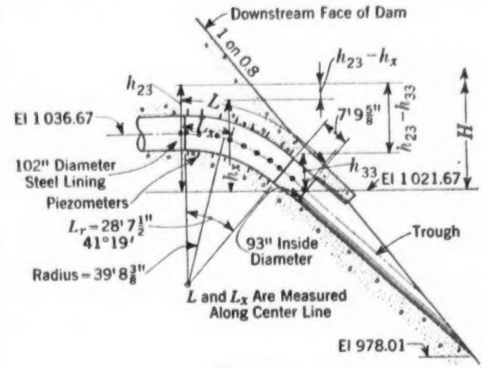


FIG. 12

Discharge Measurements.—The model and prototype discharge comparison is indicated in Fig. 13. Curve 1 is based on the 1 : 17 scale model tests for which $Q_p = Q_m \cdot L^{2.5}{}_r$, whereas Curve 2 is the model rating curve corrected to take into account the difference between model and prototype friction. To make this correction, it was first necessary to estimate an average friction coefficient for the prototype outlet. This was not readily accomplished because of a lack of reliable friction-coefficient data at Reynolds numbers as high as 5×10^7. Nevertheless, it was estimated that a value of 0.0085 for the friction factor would be reasonably accurate. The model friction coefficients and Reynolds numbers were then computed from the laboratory pressure tests and plotted in Fig. 6. Since the model friction is too large, the model heads for given discharges would be too high. Accordingly, assuming that all losses may be scaled to the prototype except friction, the model heads were reduced by the value $(f_m - f_p)\,\dfrac{L}{D} \times \dfrac{V^2}{2\,g}$, in which f_m was obtained from Fig. 6 and $f_p = 0.0085$ as estimated. After the pressure data for the prototype became available, the friction coefficients and Reynolds numbers were computed and plotted in Fig. 6. The average value for f_p was 0.0095 as compared to an estimate of 0.0085. Two 40-ft reaches were used for the prototype computations, starting at a distance of approximately eight diameters downstream from the entrance of the outlet, corresponding reaches being used for the model computations. That this is probably not a sufficient length to develop a normal velocity distribution is seen in Fig. 6, where the upper points were obtained using the first 40-ft reach, whereas the lower points were obtained from the following 40-ft reach.

The discharges given by Curve 3, Fig. 13, which were used for computing the prototype friction factors and Reynolds numbers, were developed from prototype pressure measurements, treating the bellmouth as a submerged ori-

fice. The average pressure heads measured at piezometer ring 11 enabled the values of h_e to be computed (see Fig. 4(*a*)). The four piezometers at this point are located 4 ft $5\frac{3}{16}$ in. from the entrance and at the tangent between the bellmouth and transition immediately downstream, as shown in section in Fig. 4. Using the area of the bellmouth at this point, and assuming a coefficient of discharge, the nozzle formula

$$Q = 0.98\, A_{11}\, \sqrt{2\, g\, h_e} \qquad (2)$$

was applied. Similar results would be obtained using h_e and the area at the start of the bellmouth, but with $C_q = 0.60$, or by applying the same area and coefficient with the head on the bellmouth reduced by the losses in the bell-

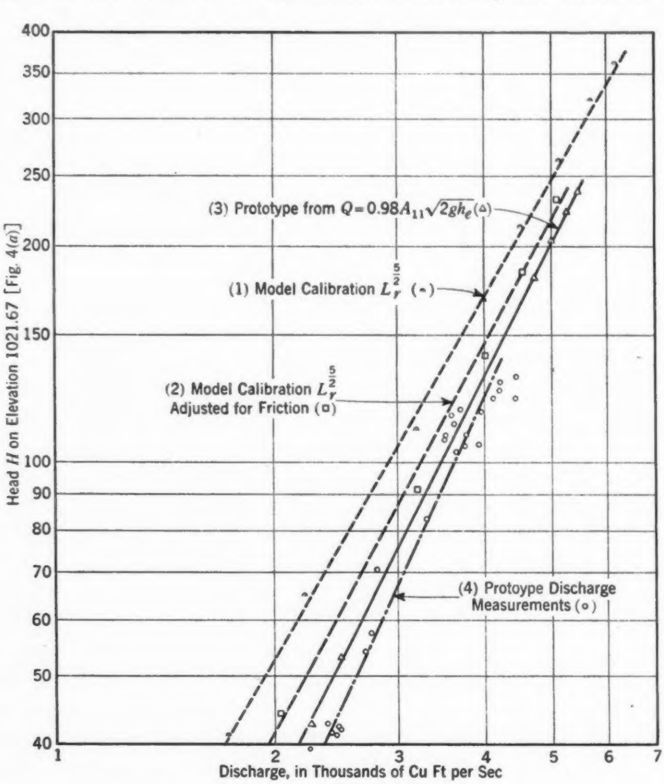


FIG. 13.—COMPARISON OF DISCHARGES

mouth instead of h_e. These computations are based on the premise that the bellmouth fits the jet from a sharp-edged orifice placed in the plane of the upstream face of the dam.

Curve 4, Fig. 13, was obtained from discharge measurements at a river gaging station one-half mile downstream from the dam. Since it is impossible to pass the flow of the river through only one outlet, the discharge appli-

cable to a single outlet had to be computed by an indirect method. First, a rating curve was obtained for one lower outlet before the water surface in the reservoir reached the intermediate outlets. This was accomplished by plotting effective head (reservoir elevation minus tailwater elevation at the gaging station) against the discharge for one lower outlet (total measured discharge divided by the number of lower outlets operating). As soon as the intermediate outlets began to operate, the total river discharge increased accordingly, but knowing the effective head on the lower outlets and the number operating, it was possible to obtain the discharge per outlet from their rating curve, and thus the total discharge contributed by the lower outlets. Subtracting this value from the measured flow in the river, and dividing by the number of intermediate outlets operating, gave the discharge for one outlet as shown by Curve 4.

This method assumes that each outlet will discharge the same amount, and that the jets from the intermediate outlets will not affect the effective head on the lower outlets. Although these assumptions are not strictly correct, it is believed that Curve 4, Fig. 13, and certainly Curve 3, would agree within 5% of any rating curve obtained by measuring the discharge of only one outlet.

In comparing the four rating curves over a range of head of from 230 ft to 50 ft, the uncorrected model discharge (Curve 1) is from 10% to 21% less than the prototype discharge computed at the bellmouth (Curve 3), whereas the corrected model discharge (Curve 2) is only 3% to 8% less. Similarly, the uncorrected model discharge (Curve 1) is from 11% to 27% less than the measured prototype discharge (Curve 4), whereas the corrected model discharge (Curve 2) is only 5% to 15% less.

Summary.—The agreement between the model and prototype in this quantitative comparison is reasonably good except in one instance. Considering experimental errors and lack of similitude relative to Reynolds' number, the pressure drop in the model bellmouth agrees with the prototype within 5%; the pressure drop in the elbow and cone of the model, on the other hand, is 30% greater than in the prototype; and, finally, the model estimate of the prototype discharge is, on an average, 8% low. Even though complete accord could not be obtained, the 1 : 17 scale model estimates are on the safe side and the field measurements show that an excellent prototype design has been evolved. It follows, therefore, although complete similitude was lacking, that the model could be relied upon to prove not only the inadequacy of several proposed designs, but also to develop the satisfactory design finally adopted.

Conclusions

To make better model-prototype comparisons, it is evident that plans should be prepared for procuring field measurements during the model tests and while the prototype is being designed. This not only enables a sufficient range of model data to be obtained, but it also makes certain that the instruments required at the prototype will be located properly and incorporated as a part of the design of the structure. Even these precautions may not

yield the desired results because of the difficulties in actually obtaining prototype flow measurements. Thus, the reasons for so few quantitative comparisons are traced to the necessity of waiting years before sufficient water is available, or to the difficulties in measuring large discharges and high velocities, or to the fact that the measured discharge is derived from several sources instead of the one to be used in a comparison. Finally, the model experiments themselves introduce objectionable factors influencing comparisons, the chief one being the inability to test the model of conduits at the correct range of Reynolds numbers. In any event it is important to build hydraulic models to as large a scale ratio as possible.

The qualitative and quantitative examples of model-prototype comparisons given in this paper reaffirm the high degree to which the model can be relied upon. Although the avowed purpose of any comparison, particularly the quantitative type, should be to affirm the theory of hydraulic models or to modify the transference relations if necessary, it cannot be accomplished from only a few comparisons. At present, the emphasis should be laid on the limitations and accuracy of quantitative estimates based on model tests. In this regard, it is equally important to show both inferior and good model-prototype comparisons. The lessons learned therefrom will certainly demonstrate the adequacy of present-day model research.

Acknowledgments

The material used for this paper represents the efforts of many individuals in the laboratory and field. The 1 : 17 scale model tests were conducted by D. M. Lancaster, Jun. Am. Soc. C. E. The planning of the prototype piezometer installations and field tests was done by C. W. Thomas and J. W. Ball, Assoc. Members, Am. Soc. C. E. The field tests were made by L. J. Snyder and A. F. Myers. D. J. Hebert contributed valuable suggestions for presenting the data. All tests were under the direction of the senior author in the materials, testing, and control division which is supervised by R. F. Blanks and Arthur Ruettgers, Members, Am. Soc. C. E., senior engineers. Design studies and investigations are under the direction of J. L. Savage, Hon. M. Am. Soc. C. E., chief designing engineer. S. O. Harper, M. Am. Soc. C. E., is chief engineer of the Bureau of Reclamation, and J. C. Page, M. Am. Soc. C. E., is commissioner of Reclamation. E. W. Lane, M. Am. Soc. C. E., at the University of Iowa, and the staff of the U. S. Waterways Experiment Station, Vicksburg, Miss., contributed many of the references that comprise the Bibliography at the end of this Symposium.

A DRY-DOCK SUCTION CHAMBER

BY C. B. PATTERSON,[4] JUN. AM. SOC. C. E.

SYNOPSIS

Other papers of this Symposium describe studies in which the analyses of model and prototype data could be based upon direct quantitative comparisons. This paper will be devoted to a study wherein, due to the lack of certain measurements in the prototype, the comparison of model and prototype performance was approached from an inductive standpoint. The problem under investigation was the unsatisfactory operation of pumps used to drain a large dry dock; the medium of study was a 1:10-scale model constructed at the Waterways Experiment Station.

The dry dock, together with its culvert system and suction chamber, is shown by the general plan drawing in Fig. 14. (Interconnecting passages to the suction chamber from adjacent dry docks are not shown.) The suction chamber is shown in greater detail in Fig. 15, and the arrangement of pump intakes (1, 2, and 3) and sluice gates (3, 4, and 5) is indicated. This arrangement, the high rate of discharge (130,000 gal per min maximum per pump), and the limited dimensions of the suction chamber, gave rise to unstable and sometimes violent flow conditions, which were manifested at the pumps by vibration, loss of capacity, and symptoms of cavitation. (All such objectionable features will be covered by the general term "rough operation.") Test runs in the prototype yielded notes and observations as to the character of pump performance, but did not provide any numerical data or evidence as to the character of flow within the suction chamber.

In constructing a model for the investigation, it was essential that the proper distribution of flow through the three sluice gates be assured. Accordingly, the entire system of side culverts and inlets was reproduced in the model. It was not feasible to reproduce the prototype pumps by means of miniature pumps of a homologous series; however, the proper metered discharges were induced through the intakes by ordinary centrifugal pumps. For purposes of visual observation, the suction chamber and pump intakes were made of transparent pyralin.

To gain some idea as to flow conditions within the suction chamber, compressed air was injected into the model under operating conditions similar to those known to give unsatisfactory pump operation in the prototype. From observations of the air bubbles in these preliminary tests it was learned (*a*) that flow entering the suction chamber from the sluice gates created initial rotation about one or another of the pump intakes (depending upon the particular conditions of operation), and (*b*) that initial vortices so formed were accentuated

[4] (Deleted by censors.)

as the water was drawn upward into the bellmouth intakes. Under such conditions it was reasonable to expect that the pressure at the axis of any intake so

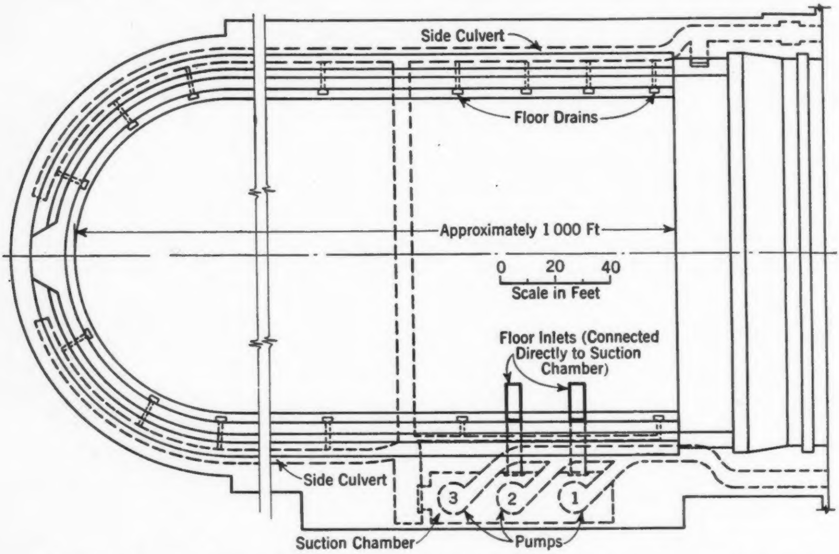


FIG. 14.—GENERAL PLAN OF DRY DOCK SHOWING CULVERT SYSTEM AND SUCTION CHAMBER]

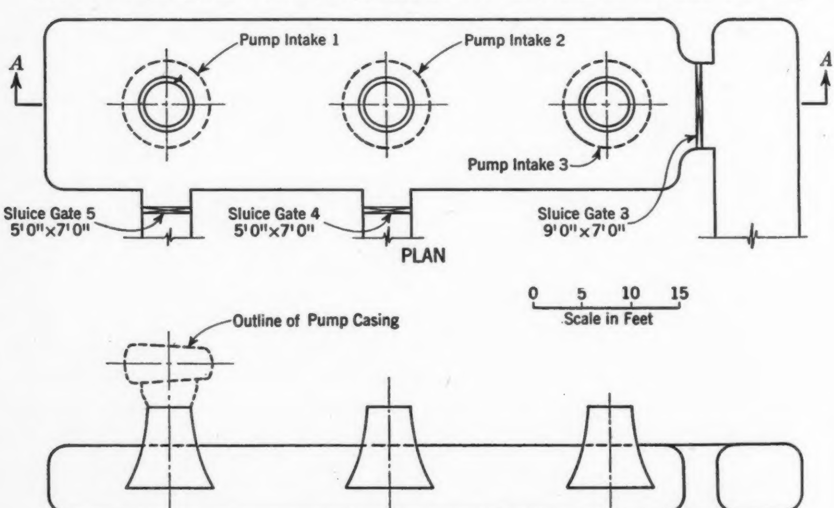


FIG. 15.—ARRANGEMENT OF ELEMENTS WITHIN A PUMP SUCTION CHAMBER

affected (that is, at the center of the vortex) would frequently fall below the suction lift capability of the pump. It was concluded that the rapid and violent fluctuations of pressure attending vortex action within the intakes were

responsible for an alternate breaking and restoring of suction head at the pumps, resulting in the aforementioned symptoms.

To support this conclusion, and to provide a basis for the comparison of model and prototype performance, it was desired that a record be made of the amplitudes and frequencies of pressure fluctuations within the pump intakes. It was believed that such data would permit the establishment of a criterion for pump operation, whereby rough pump operation would be revealed by a record showing fluctuations of great amplitude and frequency, and whereby smooth operation would be indicated by slight variations of pressure. From such a criterion, the model reproduction of prototype performance could be confirmed, and the effectiveness of any corrective measures under trial could be evaluated.

To obtain a record of pressures within the suction bells, the instrument shown in Fig. 16 was devised. The operation of this instrument is very simple: Pressure impulses originating at the pitot-tube opening are transmitted through

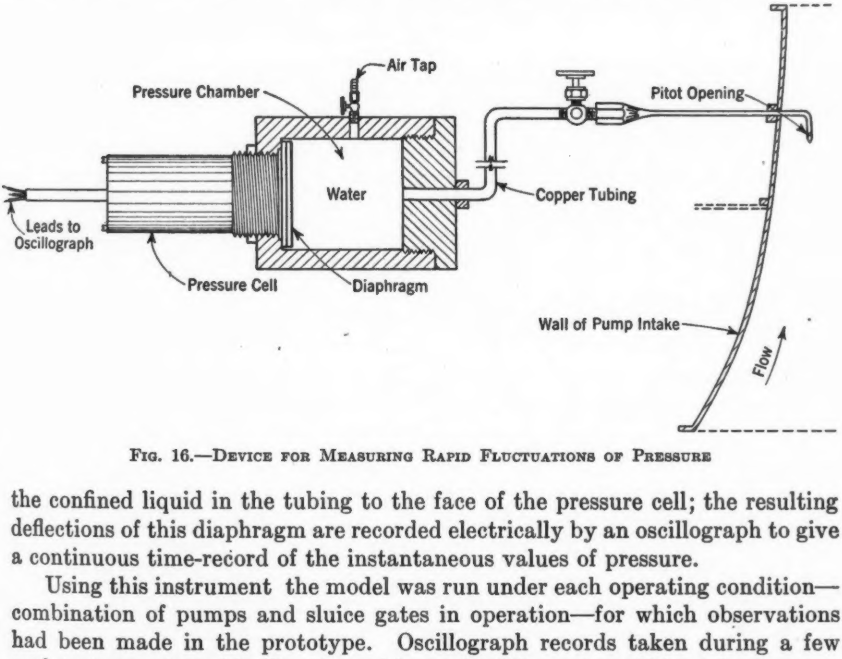


FIG. 16.—DEVICE FOR MEASURING RAPID FLUCTUATIONS OF PRESSURE

the confined liquid in the tubing to the face of the pressure cell; the resulting deflections of this diaphragm are recorded electrically by an oscillograph to give a continuous time-record of the instantaneous values of pressure.

Using this instrument the model was run under each operating condition—combination of pumps and sluice gates in operation—for which observations had been made in the prototype. Oscillograph records taken during a few such tests are plotted in Fig. 17; and for each of the test conditions the observations made under comparable conditions in the prototype are recorded in Table 1. (In Fig. 17, and all other figures, dimensions are expressed in terms of prototype equivalents.) By comparison of model and prototype observations it may be seen that they are consistent in all cases. Furthermore, the criterion for rough and smooth pump performance is definitely established. From such a model-prototype comparison it was safe to assume that predictions as to the effectiveness of corrective measures could be relied upon.

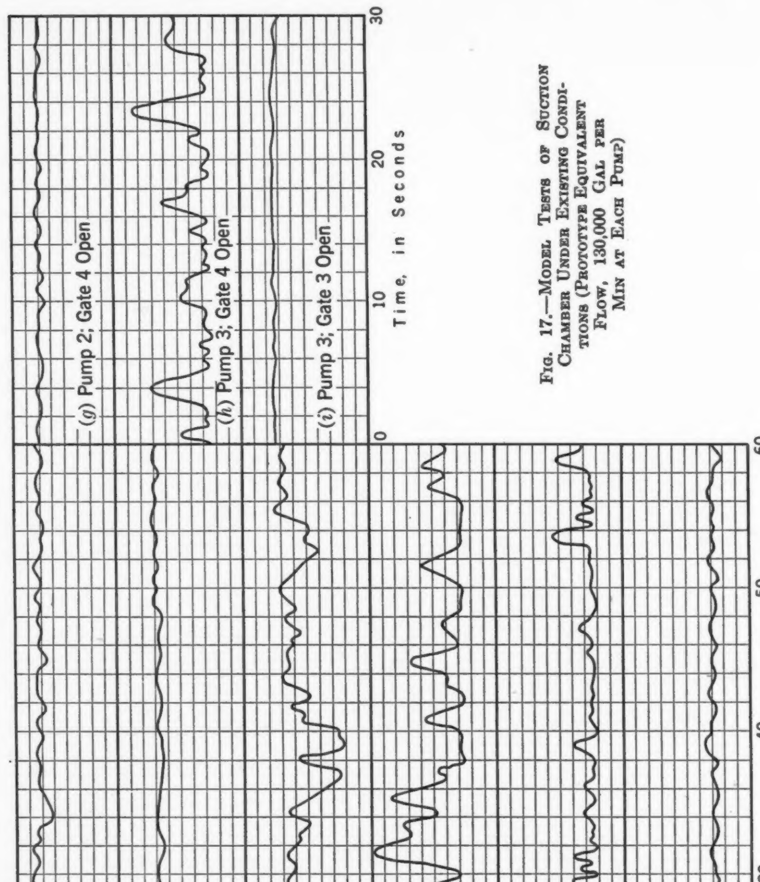


FIG. 17.—MODEL TESTS OF SUCTION CHAMBER UNDER EXISTING CONDITIONS (PROTOTYPE EQUIVALENT FLOW, 130,000 GAL PER MIN AT EACH PUMP)

It was considered that the unsatisfactory performance of the pumps would be improved if the vortices in the intakes could be broken up. For this purpose, vertical vanes of the design shown by Fig. 18 were installed in the model pump intakes. In order that flow in a longitudinal direction be unobstructed,

TABLE 1.—TEST CONDITIONS FOR MODEL OBSERVATIONS IN FIG. 17
(Discharge 130,000 Gal per Min)

Fig. 17	Sluice gates open	Pump operating	Corresponding prototype observation
a	3, 4, and 5	1	....
b	3, 4, and 5	2	....
c	3, 4, and 5	3	Operation very rough
d	3	1	Extremely bad operation
e	3	2	Generally unsatisfactory operation
f	3	3	With more than one pump operating on Sluice Gate 3 the farthest unit could not operate; generally unsatisfactory operation
g	4	2	Under initial conditions Pump 2 operated satisfactorily
h	4	3	Operation very rough
i[a]	3	3	When Pump 2 was shut down, power input to Pump 3 decreased. When Sluice Gate 3 was opened power input to Pump 3 decreased again and its operation was good.

[a] Initially (see Figs. 17(*g*) and 17(*h*)) pumps 2 and 3 were each discharging 130,000 gal per min with Sluice Gate 4 open. Then (see Fig. 17(*i*)) Pump 2 was shut down and Sluice Gate 3 was opened.

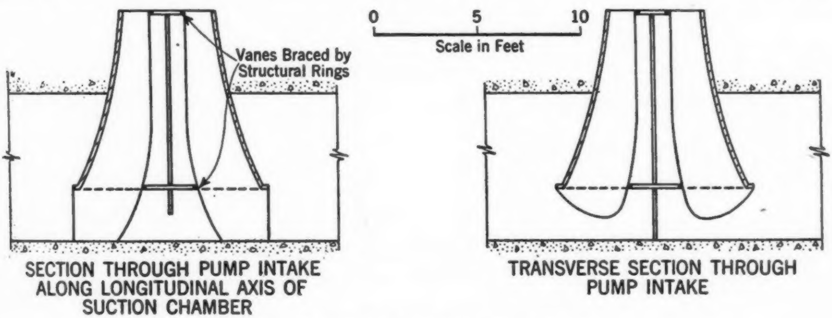


FIG. 18.—DESIGN OF VANES FOR PUMP INTAKES

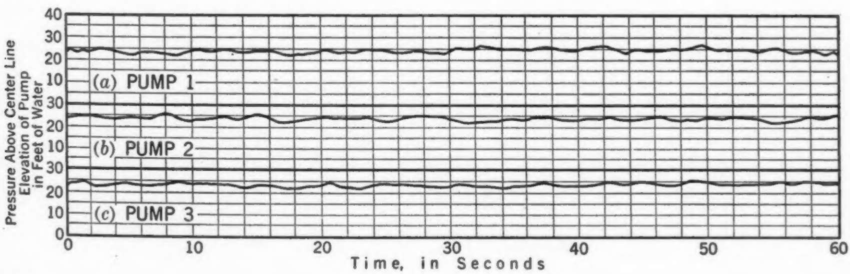


FIG. 19.—MODEL TESTS FOLLOWING THE INSTALLATION OF VANES IN PUMP INTAKES (PUMPS 1, 2, AND 3 EACH DISCHARGING 130,000 GAL PER MIN; SLUICE GATES 3, 4, AND 5 OPEN)

transverse vanes were not extended to the floor of the model suction chamber. In the prototype, however, all vanes were extended to the floor. The results

of a model test run with these vanes in place are shown in Fig. 19. By comparing these results with those of Fig. 17, the improvement is evident, and a smooth operation of the pumps is indicated. Modifications of the basic design vanes were also tested, but no improvement over the basic design was found.

Accordingly, vanes of the basic design were adopted for installation in the prototype. As a final step in the analysis, the model predictions were thoroughly substantiated by the prototype; it is reported that operation of the pumps, following installation of vertical vanes in the intakes, has been entirely satisfactory. A complete model-prototype confirmation would include the measurement of pressure fluctuations in the prototype but thus far such measurements have not been practicable.

INTERESTING LOCK CONSTRUCTION

By Martin E. Nelson,[5] M. Am. Soc. C. E., and
James J. Hartigan,[6] Esq.

Synopsis

A lock and dam, one of the major projects constructed on an eastern river by the Tennessee Valley Authority (TVA), was designed in 1934. A model of the proposed lock was built and tested at the U. S. Engineer Sub-Office, Iowa City, Iowa, for the purpose of aiding in the hydraulic design of the filling and emptying system. As the Tennessee development continued, other locks were designed and models were used to study special problems and to check the final plans.

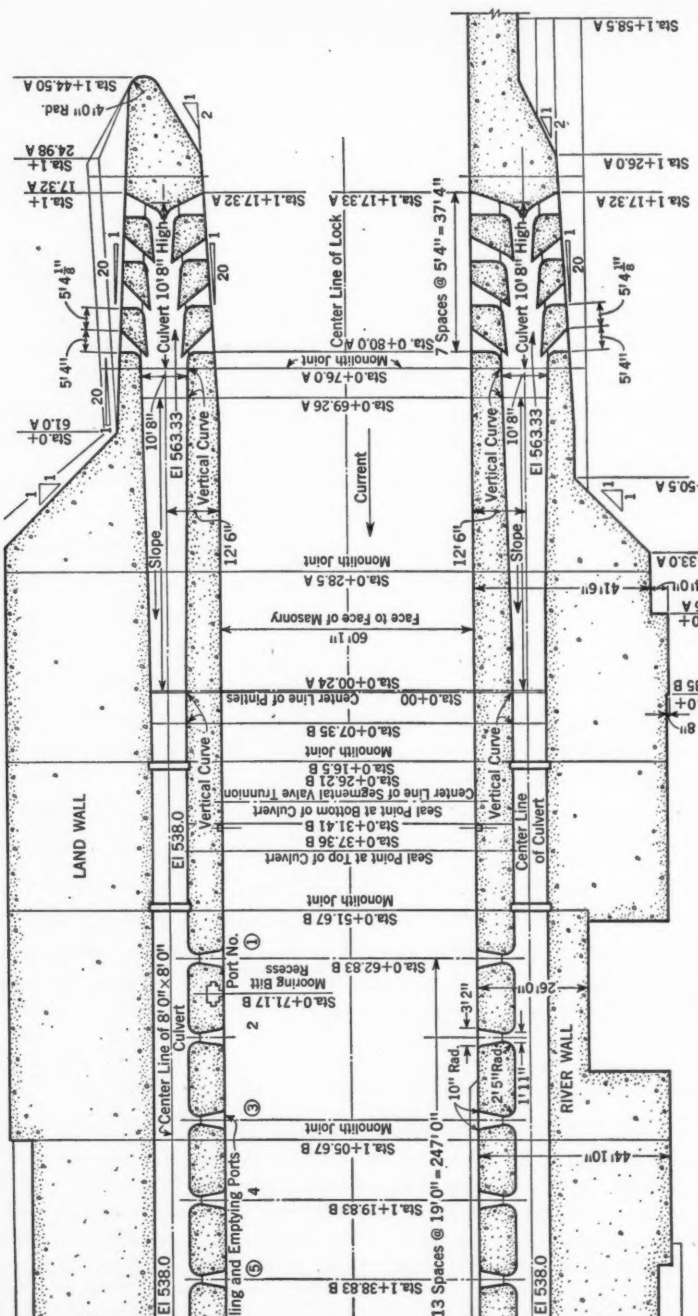
The question of the reliability of hydraulic lock models in simulating conditions which would exist in the full-size structures developed with the increased use of this method of design. Accordingly, steps were initiated to test a model of an existing lock in which exact geometric similarity could be obtained. The project chosen for the correlation was a lock, the first project completed in the river by the TVA. A model of the lock was built in December, 1936, and comparisons were made between model tests and a limited amount of prototype test data available at that time. The results of these comparisons were satisfactory, but the prototype data available were not adequate to warrant drawing definite conclusions.

The division engineer of the Ohio River Division initiated steps for testing a number of prototype locks in two eastern rivers to verify the model tests and to study, under actual operating conditions, the hydraulics of various modifications of the port and culvert system. Tests were made on four locks on one river, and at another eastern river lock. Model tests had been made on only three locks, and this paper will be confined to a discussion of tests made on these three locks and their models.

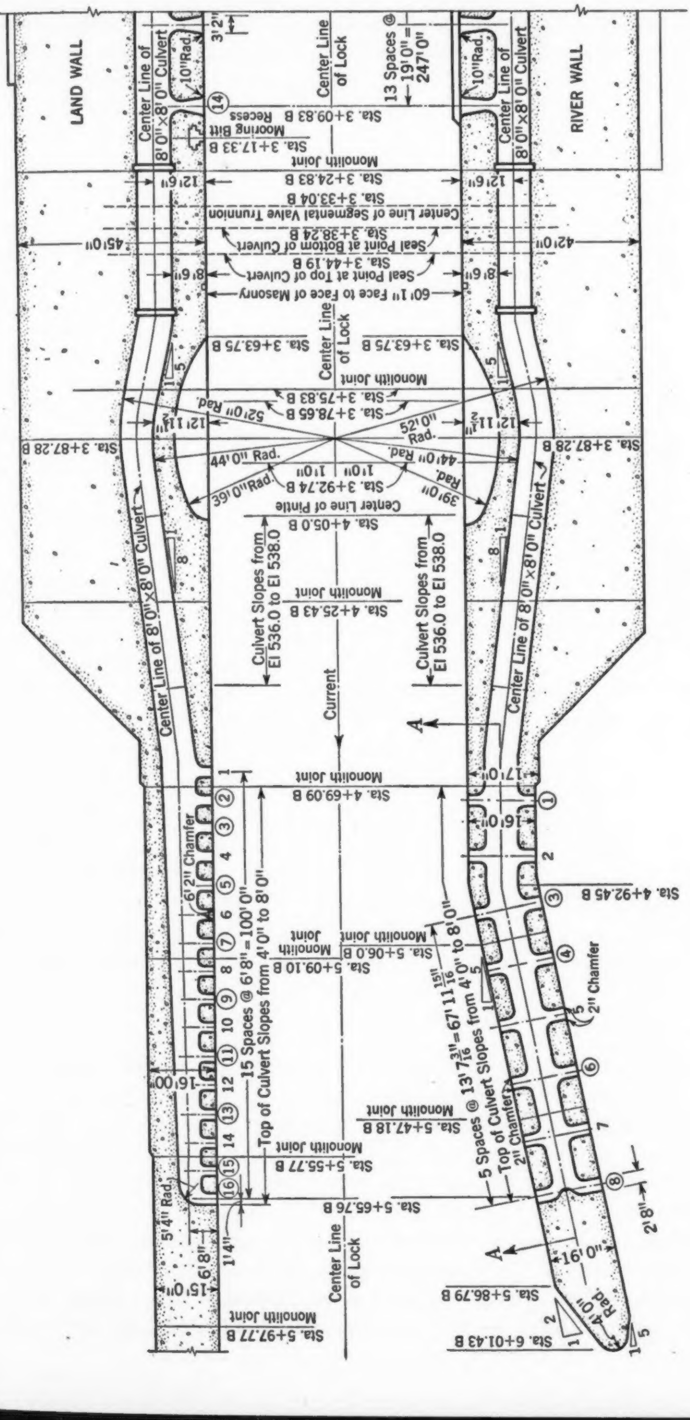
General Description of Lock Hydraulics Systems

The locks in question are all equipped with the same general type of filling and emptying system. Longitudinal culverts located in the side-walls of the lock open into the upper pool through gallery intakes and into the lower pool through similar discharge sections. Short lateral ports connect the culverts with the lock chamber near the floor. The flow in each culvert is controlled by valves upstream from the first and downstream from the last lateral port

[5] (Deleted by censors.)
[6] (Deleted by censors.)



(a) LOCK WALLS AND CULVERTS, UPPER END



(b) LOCK WALLS AND CULVERTS; LOWER END

FIG. 20.—PROJECT LOCK

to the chamber. In filling the lock chamber the downstream valves remain closed, the upstream valves are opened at a prescribed rate, and water from the upper pool enters the culverts through the intake galleries and flows into the lock chamber through the short lateral ports, the flow of water continuing until the water levels in the chamber and upper pool are equalized. To empty the lock the upper valves are closed, the lower valves opened, and water in the chamber discharges into the lower pool until their levels coincide.

Fig. 20 shows the essential details of the hydraulic system of one lock. The intake galleries and culverts are designed to effect as uniform acceleration of the flow as possible from the face of the lock wall to the valve section. The ports (see Fig. 21) were streamlined with well-rounded entrance corners

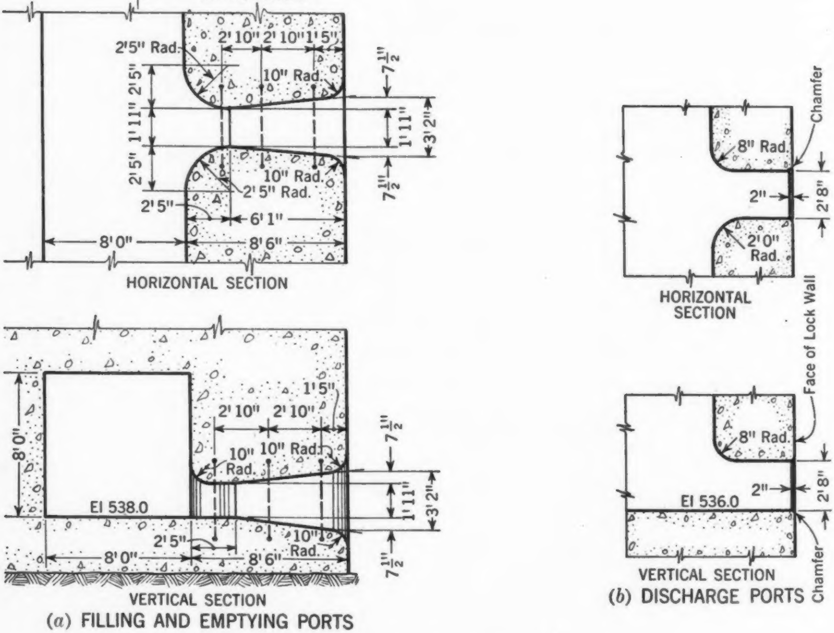


FIG. 21.—SECTIONS THROUGH LOCK PORTS

to eliminate separation of the jets from the leeward walls. The bores were tapered to effect greater discharge capacity and to reduce turbulence in the chamber by expanding the jets and decreasing their velocity. The discharge section was also designed with a view to obtaining uniform distribution of discharge in the lower lock entrance and to minimize turbulence in the immediate area.

Another lock has a system essentially identical, only enlarged in scale in approximate proportion to the relative dimensions of the structures.

The hydraulic system of still another lock is typical of the designs used in inland waterways locks prior to the use of models in analyzing the hydraulics

involved in their operation. Deviation from perpendicular lines to effect better hydraulic efficiency was practically avoided. Simplified construction was given primary consideration at the expense of efficiency throughout a lifetime of operation.

Scope of the Tests

The parallel tests that were made in corresponding model and prototype locks will be described in general terms in order to eliminate the necessity of discussing the experimental work performed on each individual structure. Each type of test considered was performed on at least one pair of similar locks and in most cases on all three pairs considered in this paper:

(*a*) The stages near both ends of the lock chamber were observed with respect to time in filling and emptying operation. From these data were plotted filling or emptying curves, rate-of-rise or rate-of-fall curves, and water-surface slope curves, all with respect to time. Comparisons of model and prototype were made on the basis of total time required to complete the operation, the coefficients of discharge for the entire hydraulic system, the maximum value of the rate of change in stage in the lock chamber, and the rate of decrease in the flow after the maximum had been reached as indicated by the slopes of the rate of rise or fall curves.

(*b*) Velocities were observed in the ports of the intake section, the manifold section, and the discharge section, and from these data comparisons of the distribution of discharge in the respective sections were made.

(*c*) Pressure measurements observed at the ends of the culvert transition sections were used in conjunction with volumetric discharge measurements in the lock chamber to determine discharge calibrations and venturi coefficients.

The primary use for hydraulic models is to provide data for design. As such, the functions of a model are generally completed before the prototype is built, and not infrequently the final design is modified from that used in the model, due to contingencies which had not arisen until the final stages of the design were reached and which were considered too insignificant to warrant the expense of checking in the model. As a result, very few prototypes are built exactly like the models from which their designs were developed. Such differences were found in the locks covered by this study, but wherever possible, corrections will be applied or dimensionless parameters used to validate the desired comparisons.

Comparison of Prototype and Model Tests

Intake Ports.—A primary consideration in the design of the intake sections of one lock was a uniform distribution of flow in the ports. With flow in one direction only, a tapered culvert section could be used. That the design successfully approximated a uniform distribution in the four ports on one side of the culvert, and that agreement existed between model and prototype distributions, are indicated by the graph in Fig. 22. The discharge in the model intake ports varied uniformly from 21.5% to 28.5% and in the corresponding prototype ports from 22% to 27%.

Transition Section.—The rating equation for a culvert transition section, expressed in terms of culvert discharge Q and pressure head differential between two sections $h^{0.5}{}_p$, is based on the Bernoulli theorem; thus:

$$Q = K\, h^{0.5}{}_p \qquad (3)$$

The coefficient of discharge for the transition section meter is equal to the ratio of the observed to the theoretical rating. A comparison of prototype and

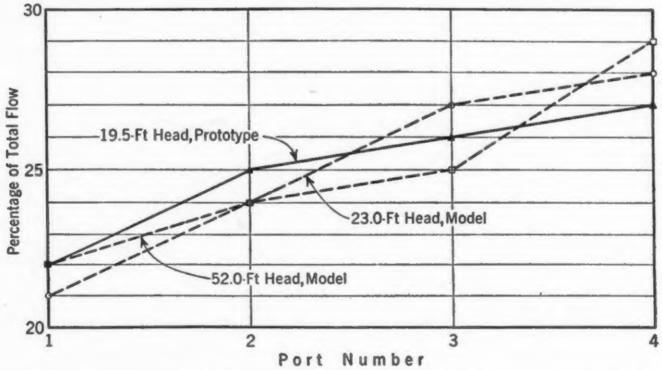


FIG. 22.—DISTRIBUTION OF FLOW THROUGH LOCK INTAKE PORTS (CONSTANT FLOW)

model locks is made, as shown in Table 2, on the basis of the performance of their culvert transition sections as flow meters. The lack of exact geometric similarity in the transition sections of model and prototype accounts for the small difference in the constants of the theoretical rating equations; however corresponding equations and coefficients for model and prototype are in virtual agreement.

TABLE 2.—COMPARISON OF MODEL AND PROTOTYPE LOCKS

Description	Rating Equation (Eq. 3)		Coefficient of discharge C
	Observed K_o	Theoretical K_t	
Model	1,400	1,500	0.93
Prototype	1,410	1,525	0.92

Lock Stage Tests.—The levels of the water surface in the lock chamber at respective time intervals during filling operations in the model and prototype are shown in Fig. 23. The points defining the filling curves are averages of two stage measurements, one near each end of the lock chamber, observed at each time interval.

The filling curves for the model and prototype locks indicate a considerable difference in the total time of operation and in the form of the filling curves. There were two factors which materially affected the results of these tests—one a difference in construction of the lock-chamber ports (Fig. 24), and the other a difference in the time of opening the culvert valves. In the model the total cross-sectional area of the lock-chamber ports at the smallest section was 125.0 sq ft per culvert, whereas in the prototype the corresponding areas

were 145.0 sq ft in the river wall and 147.8 sq ft in the land wall. The culvert valves in the model were opened in a time corresponding to 2 min in the prototype, whereas in the filling test shown the right valve of the prototype required 5 min 4 sec to open, and the left valve 6 min 37 sec. A third factor which

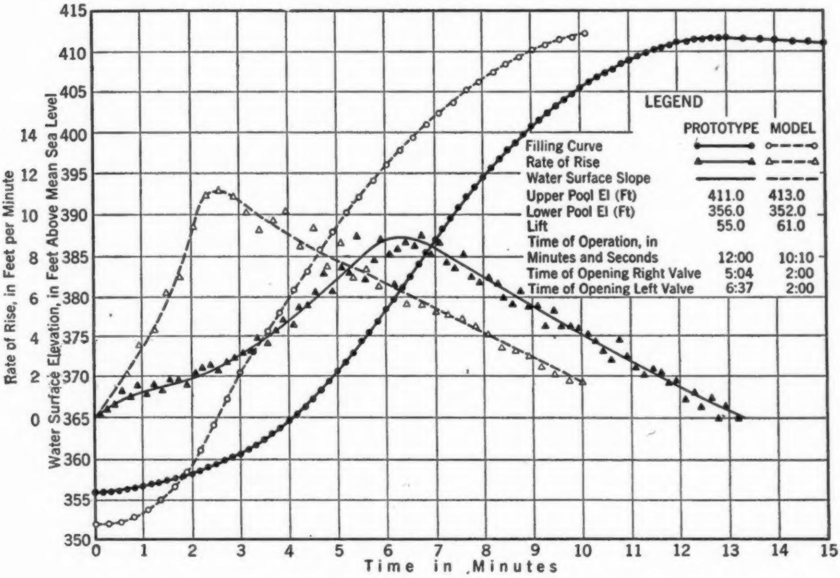


FIG. 23.—FILLING CHARACTERISTICS

also affected the filling time was a difference in the lifts. In the model a lift of 61 ft was simulated, whereas in the prototype 55 ft was the maximum difference in water levels existing during the period of the tests.

If the two filling curves shown in Fig. 23 were matched at the instant when the corresponding locks were full, it is noted that they would follow approximately the same path for the period after the valves were completely open. The larger throat area in the prototype lock-chamber ports, which tended to increase the efficiency of the prototype over that of the model, appears to have been offset for the most part by the larger and more efficient divergence angle in the model ports.

The maximum rate of rise or fall of the water level in a lock occurs shortly after the culvert valves are fully open. Because of the difference in time of valve operation, the rate-of-rise and rate-of-fall curves for the model and prototype of one of the locks differ considerably. The maximum rate of rise in the model was 11.1 ft per min at 2 min 35 sec, and in the prototype it reached 9.0 ft per min at 5 min 25 sec. In a manner similar to the filling curves, the rate-of-rise curves for model and prototype follow nearly parallel paths after the peak rate has been reached; the average slope of the curve for the prototype is 1.40 ft per min², while that for the model is 1.24 ft per min².

The differences in construction details and valve operation, noted in the discussion of the filling tests, apply also with respect to the emptying tests. However, the larger port area and smaller lift existing in the prototype, tending to reduce the emptying time, were practically offset by the longer valve period. The emptying time in the prototype was 13 min 18 sec, whereas in the model it was 13 min 29 sec. The dissimilarities in construction details of the two structures make comparisons of the filling and emptying characteristics unsatisfactory.

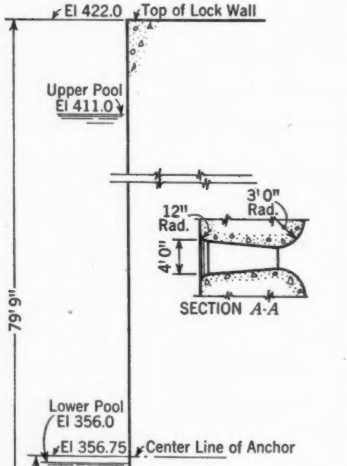
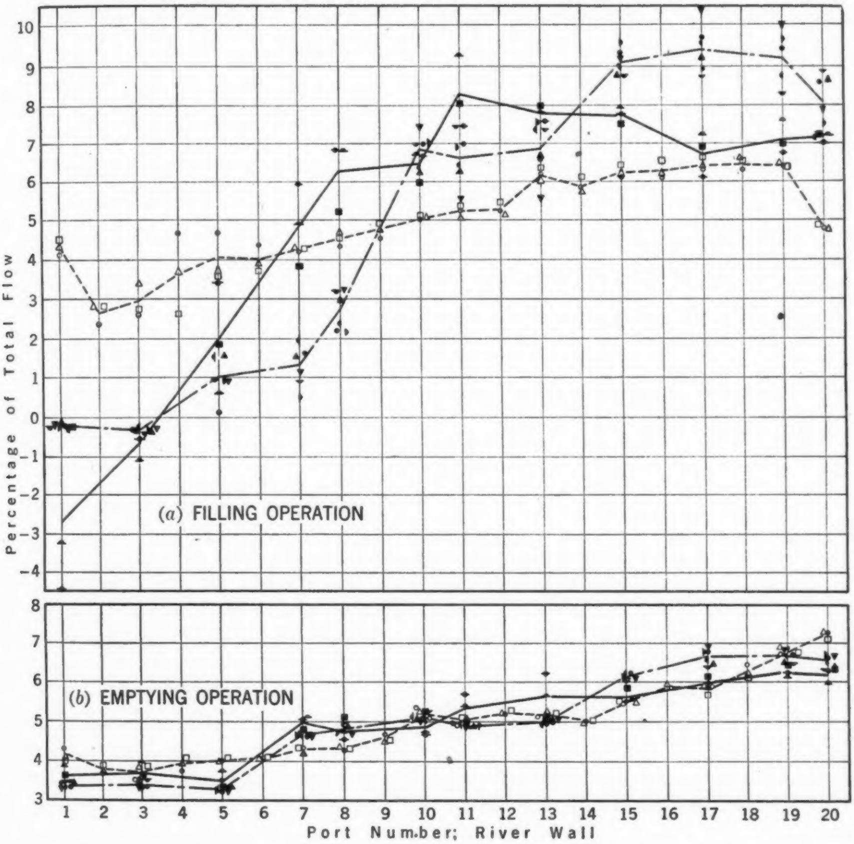


FIG. 24.—SECTIONS THROUGH CENTER LINE OF LOCK-CHAMBER PORT (20 PORTS IN EACH WALL)

Lock-Chamber Ports.—Model tests have indicated that longitudinal slope of the water surface in the lock, such as was created by surges or by the energy change from kinetic to potential accompanying the decrease in velocity of flow from one end of the lock to the other, is the primary factor that influences movement of a vessel during lockage. Improper operation of the valves and uneven distribution of discharge in the ports of a side culvert system will promote surging in the lock; and, once started, surging will continue throughout a considerable part of the filling period. It is important, therefore, that the filling controls be operated so as to eliminate a sudden inrush of water at the beginning of the filling period, and that the system be designed to distribute the lockage inflow evenly among the lock-chamber ports.

From the port velocity tests made in the model and prototype locks, distribution-of-flow graphs for the ports in the right or river wall for the filling operation were plotted as shown in Fig. 25(*a*). The flow distributions in the model and prototype differed considerably, due largely to the dissimilarity of the ports. In earlier laboratory experiments it was found that the distribution of discharge from the ports during a filling operation was influenced to a large extent by the value of the cumulative cross-sectional area of the lock-chamber ports at the smallest section. When the port area exceeded a certain value, the shape of the hydraulic gradient was changed so that during part or all of the filling period water would flow from the lock chamber into the culvert through a number of the ports at the upstream end of the manifold. The prototype port area exceeds that of the model by 16% in the river wall and 18.2% in the land wall. Excess area in the prototype is equivalent to that of three ports. The effect of the excess port area in the prototype is indicated in Fig. 25(*a*). Discharge through ports 1 and 3 was reversed, flowing from the lock chamber into the culvert during the entire period of the filling

operation, and, although the flow in port 2 was not measured, it undoubtedly was negative also. Therefore, the distribution patterns in the model and prototype manifolds cannot be correlated.



Description	Average of Three Heads[b]			Average of Six Heads[b]						Average of Three Heads[c]		
	-●-	▲	■	▼	▲	◀	●	▶	▼	□	△	○
Head, in feet............	50	45	40	35	30	25	20	15	10	40	30	22
Discharge,[a] in Cubic Feet per Second:												
(a) Filling operation....	..	..	..	6,080	5,630	5,200	4,650	4,030	3,320	6,310	5,290	4,640
(b) Emptying operation.	..	..	..	4,370	4,430	4,220	3,770	3,270	2,670	4,670	4,050	3,470

[a] Discharge refers to flow into, and out of, lock chamber through the ports in the river wall. [b] Prototype. [c] Model.

FIG. 25.—DISTRIBUTION OF FLOW, LOCK-CHAMBER PORTS

A very close similarity in the flow distribution in the model and in the prototype during emptying operations is shown in Fig. 25(*b*). The distribution of flow through the ports was only slightly affected by variations in head, and the discrepancies in port dimensions in the prototype had less effect in the emptying than in the filling operation, undoubtedly because the flow in the venturi-shaped ports was converging instead of diverging.

Velocity in Discharge Ports.—The discharge sections of one of the locks, with the exception of slight differences in a few dimensions, were constructed like the model. Flow conditions during the first part of the emptying operation differed in the two structures, but after the valves were opened completely, distribution of the flow from the ports was found to be quite similar.

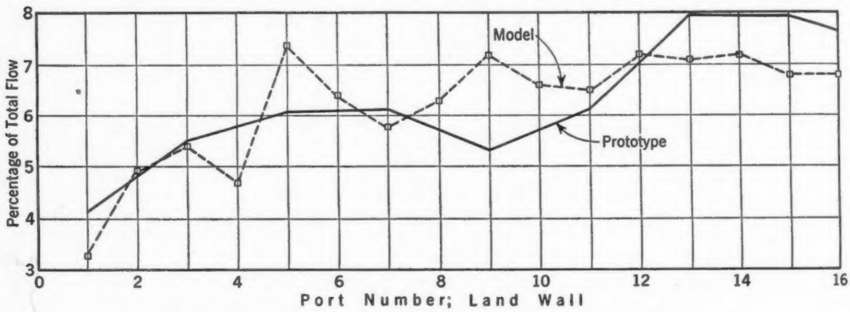
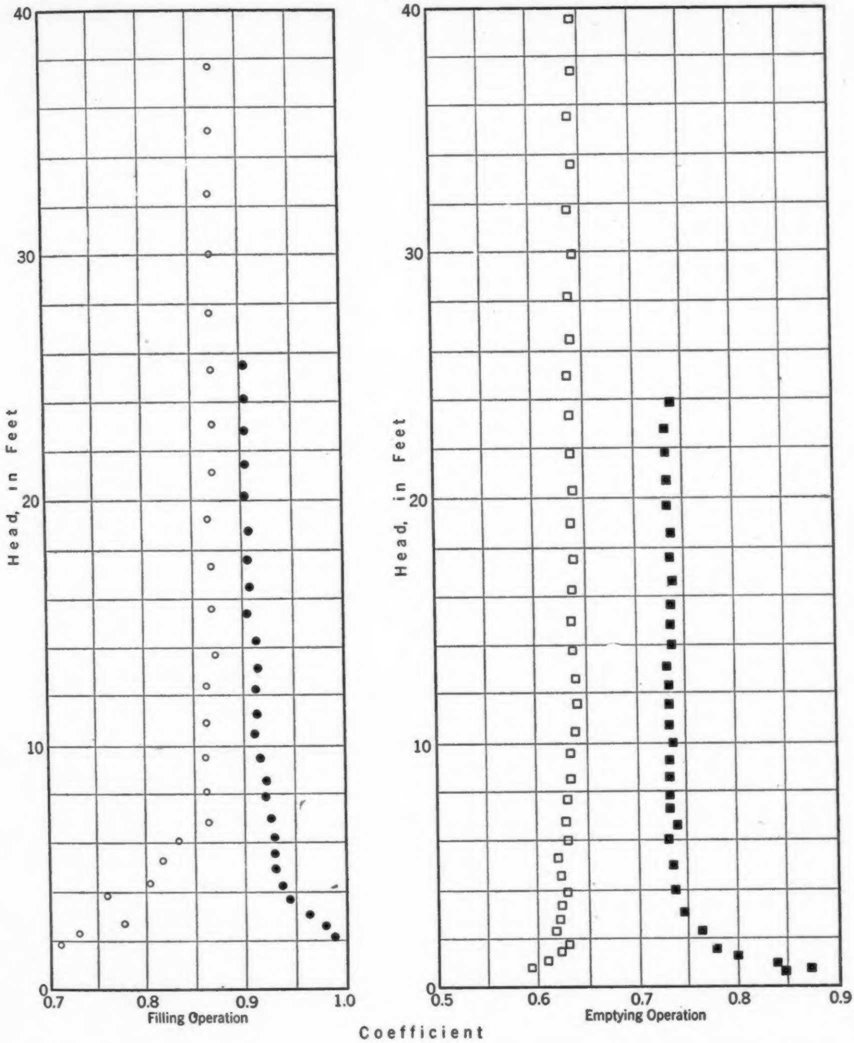


FIG. 26.—DISTRIBUTION OF FLOW, DISCHARGE PORTS (EMPTYING OPERATION)

In Fig. 26 is shown the distribution of flow from the discharge ports in the land wall of the model for a head of 39 ft. The curve for the prototype shows an average distribution for four heads (25 ft, 21 ft, 15 ft, and 10 ft), all of which occurred after the valves were open and flow conditions were relatively stable.

These curves for model and prototype are similar in having the same general trend of larger discharges in the downstream ports. The increase in the discharge for each successive port downstream is not so orderly in the discharge section (which has a culvert of variable area) as it is in the lock-chamber section which has uniformly spaced ports and constant area culvert. The percentages of culvert flow carried by the eight upstream and eight downstream ports of the discharge section were 44.4% and 55.6% for both model and prototype, variations between model and prototye having balanced in each half of the section.

Lock Coefficients.—The coefficients for lock hydraulic systems presented in this paper were computed for the period after the valves were fully opened and were based on the minimum culvert area. In the prototype lock system the section having the least area through which the water passes, both in filling and emptying, is the 12-ft by 12-ft culvert below the transition section. In the model the section of minimum area was at the throat of the lock-chamber ports, this area being 250 sq ft as compared with the minimum culvert area of 288 sq ft. The culvert area in the prototype was the same as in the model, but the cumulative port area at the minimum section was 292.8 sq ft.



Description	Legend: ●	○	■	□
Filling operation	Prototype	Model	....	....
Emptying operation	....	....	Prototype	Model
Average coefficient C (Eq. 4)[a]	0.91	0.87	0.73	0.64

[a] For heads greater than 6 ft.

FIG. 27.—LOCK COEFFICIENT

The so-called lock coefficient C_l, which is a bulk coefficient representing the efficiency of the lock, is equal to the ratio of the observed discharge to the theoretical discharge computed by the plain orifice equation using instantaneous heads:

$$C_l = \frac{v_l\, A_l}{A_c\, \sqrt{2\, g\, h}} \qquad (4)$$

In Eq. 4: v_l = the lock-chamber rate of rise or fall; A_l = lock-chamber area; and A_c = cross-section area of two culverts. The filling and emptying coefficients determined for the prototype and model locks are shown in Fig. 27 and Table 3.

TABLE 3.—COMPARISON

Item No.	Structure	Lift (ft)	Time (Min): Valve	Time (Min): Operation	Rate[a] (ft per min)	Slope[b] (ft per 100 ft)	Area (Sq Ft): Lock chamber	Area (Sq Ft): Culvert	Area (Sq Ft): Port: Throat end	Area (Sq Ft): Port: Lock-chamber end
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	(a) FILLING OPERATION									
	1st Lock									
1	Model	61	2.0	10.17	11.1	0.148	74,250	288	250	640
2	Prototype right	55	5.07	12.00	8.9	0.062	74,250	288	292.8	640
3	Prototype left		6.62							
	2nd Lock									
4	Model	50	1.00	7.20	12.6	0.148	24,300	160	240	240
5	Prototype	50	1.07	7.08	12.4	0.273	24,300	160	240	240
	3d Lock									
6	Model	39	1.00	6.15	12.5	0.117	24,300	128	102.8	211.8
7	Prototype	39	1.12	5.50	12.7	0.118	24,300	128	102.8	280.8
	(b) EMPTYING OPERATION									
	1st Lock									
8	Model	61	2.0	13.48	8.6	0.045	74,250	288	250	640
9	Prototype right	55	3.97	13.30	7.3	0.030	74,250	288	292.8	640
10	Prototype left		5.12							
	2d Lock									
11	Model	50	1.00	8.03	11.7	0.059	24,300	160	240	240
12	Prototype	50	1.07	7.37	11.8	0.106	24,300	160	240	240
	3d Lock									
13	Model	39	1.00	7.60	9.3	0.029	24,300	128	102.8	211.8
14	Prototype	39	1.12	7.60	8.8	0.052	24,300	128	102.8	280.8

[a] Maximum rate of rise or fall. [b] Maximum water surface slope. [c] Distribution based on equal number

The average coefficient for filling the prototype lock based on the culvert area, which is the smallest section in the course of flow, was 0.91 as shown in the legend. In the model the coefficient based on the culvert area was 0.87. However, the smallest area in the course of flow in the model was at the throat of the ports, and when the model coefficient is correlated to the prototype value by applying the ratio of culvert to port area, 1.15, it is increased to

1.00, which is not in agreement with the prototype value. It would appear that the venturi type flow in the ports during filling makes a comparison of the lock coefficients unsatisfactory because the divergence effect in the model and prototype was not the same.

The average coefficients for the emptying operation, based on the least culvert or port area, are for the prototype 0.73 and for the model 0.64 $\times$ 1.15, or 0.74, which are in agreement within 2%. The flow through the lock-chamber ports during an emptying operation is not similar to that in a flaring tube, and the decrease in the efficiency of the model as indicated by the lock coefficient is directly proportional to the reduction in the area of the controlling section.

OF MODEL AND PROTOTYPE

COEFFICIENTS		DISTRIBUTION OF FLOW (PERCENTAGES)[c]						Scale ratio	Item No.
		Intake Ports		Lock-Chamber Ports		Discharge Ports			
Lock hydraulic system	Transition section	Upstream half	Downstream half	Upstream half	Downstream half	Upstream half	Downstream half		
(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	
(a) FILLING OPERATION									
0.87	0.93	45.5	54.5	40.4	59.6	....	....	1	1
0.91	0.92	47.0	53.0	17.3	82.7	....	....	30	2, 3
0.62	....	....	....	31.3	68.7	....	....	1	4
0.61	....	....	....	32.4	67.6	....	....	20	5
0.88	0.93	45.5	54.5	47.0[d] / 46.5	53.0[d] / 53.5	....	....	1	6
0.95	0.95	46.0	54.0	40.8	59.2	....	....	20	7
(b) EMPTYING OPERATION									
0.64	....	....	....	42.1	57.9	44.4	55.6	1	8
0.73	....	....	....	40.5	59.5	44.4	55.6	30	9, 10
0.56	....	....	....	30.2	69.8	....	....	1	11
0.58	....	....	....	35.0	65.0	....	....	20	12
0.63	....	....	....	44.3	55.7	44.4	55.6	1	13
0.63	....	....	....	46.5	53.5	43.6	56.4	20	14

of ports upstream and downstream. [d] Model test at Norris Laboratory.

In the orifice equation, used in computing lock-chamber discharge,

$$Q = C\,A\,\sqrt{2\,g\,h} \qquad (5)$$

the head, h, for filling, is the upper pool elevation minus the lock-chamber elevation, and, for emptying, the lock-chamber elevation minus the lower pool elevation. This head is simply the hydrostatic head and is not the complete

effective head acting on the column of water in the culverts. Considering the filling operation, the lock-chamber water surface does not stop when it reaches upper pool elevation but rises above it. Therefore, at the instant when the lock is full the effective head is not zero but has some positive value. When the dynamic head factor is included in the equation for discharge the resulting coefficients for decreasing heads become smaller instead of larger. Thus, in the prototype tests, as shown in Fig. 27, the ascending value of the coefficient as the head decreases can be attributed to the fact that the effective head was not used in the computation, whereas in the model tests the decrease

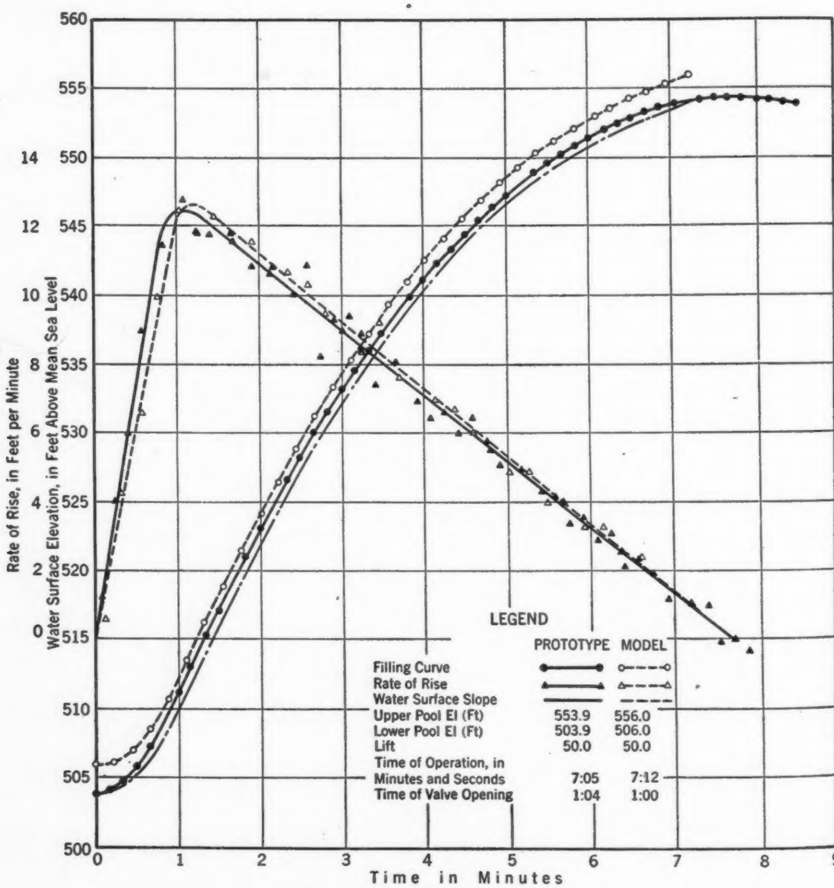


FIG. 28.—FILLING CHARACTERISTICS

in the coefficients for low heads may indicate that the hydrostatic head is more nearly equal to the effective head in the model than in the prototype.

Coefficients based on the effective head (that is, static plus dynamic) and coefficients based on static head alone have the same adverse characteristic of

not being entirely constant throughout the lock operation. On the other hand, the effective head is not so easily measured as the static head. In view of the difficulties of establishing the exact value of the effective head and of the questionable advantage, in the comparison of model and prototype, of the coefficient thus obtained over that based on the static head, all lock coefficients presented in this paper are based on the static head alone.

Wheeler Lock—Comparison of Prototype and Model Tests

Lock Stage Tests.—Stages measured with respect to time during filling operations in the Wheeler model and prototype lock chambers are shown in Fig. 28. The points which define the curves are averages of two stage measurements, one near each end of the lock chamber. Typical sections through the lock-chamber ports are shown in Fig. 29.

In the prototype, it was necessary to make the tests with existing elevations of 553.9 for the upper pool and 503.9 for the lower pool; tests had been made previously in the model with elevations of 556.0 and 506.0, respectively. In each case the head was 50 ft, but the pool levels used in the model were 2.1 ft higher than in the prototype. In Fig. 28 the filling curve of the model is shown as observed and also adjusted to agree with the lower pool elevation of the prototype. The prototype lock filled in 7 min 5 sec, whereas the model filled in 7 min 12 sec. Maximum rates of rise were 12.4 and 12.6 ft per min, and the slopes of the rate of rise curves were 1.90 and 1.94 ft per min², respectively.

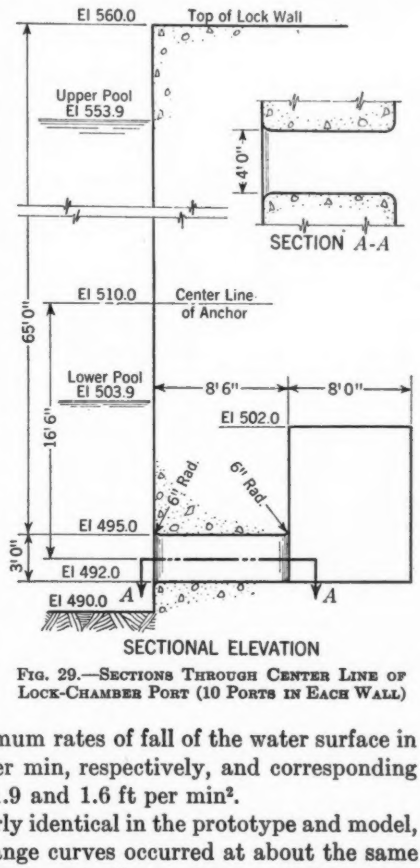


Fig. 29.—Sections Through Center Line of Lock-Chamber Port (10 Ports in Each Wall)

The emptying curve of the model was similarly observed and adjusted to agree with the upper pool elevation of the prototype. Emptying time in the prototype was 7 min 22 sec, and in the model was 8 min 2 sec. Maximum rates of fall of the water surface in the chambers were 11.8 and 11.7 ft per min, respectively, and corresponding slopes of the rate of fall curves were 1.9 and 1.6 ft per min².

The valve opening periods were nearly identical in the prototype and model, and the crests of the water surface change curves occurred at about the same time in the two structures. In the filling tests the model was slightly more efficient than the prototype, whereas in the emptying tests the reverse was true.

Velocity in Lock-Chamber Ports.—Distribution of flow in the lock-chamber ports for the left or river wall of the prototype and for the right wall of the

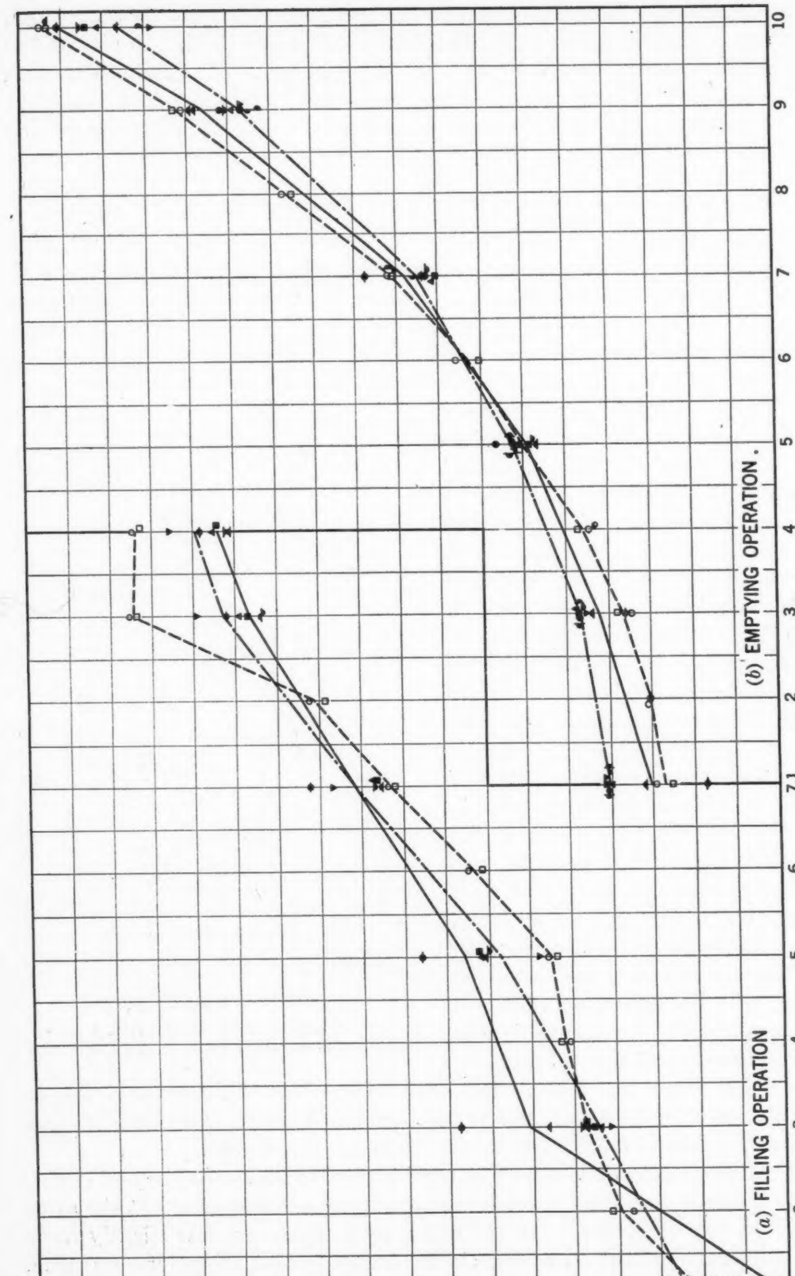


FIG. 30.—DISTRIBUTION OF FLOW, LOCK-CHAMBER PORTS (RIVER WALL) (SEE TABLE 3A)

model is shown in Fig. 30(*a*) for the filling operation. The curves for the model were based on velocity tests with steady flow through the system, whereas the curves for the prototype were based on velocity measurements made during filling operations. The distribution of flow observed in the lock-chamber ports of the model is in good agreement with that of the prototype (see Table 3*A*). Inflow into the lock through the upstream five ports in the prototype was 32.4% of the total, and in the model 31.3%. The corresponding values for the downstream five ports were 67.6% and 68.7%, respectively.

TABLE 3*A*.—Distribution of Flow, Lock-Chamber Ports (River Wall) (See Fig. 30)

Description	Average of Three Heads[b]			Average of Six Heads[b]						Average of Two Heads[d]	
	-●-	▲	▼	■	▲	●	◀	▶	▼	□	○
Head, in feet	48	45	40	35	30	24	20	15	10	35	24
Discharge,[a] in Cubic Feet per Second:											
(*a*) Filling operation	..	..	2,430	2,280	2,110	1,920	1,750	1,540	1,260	2,370	1,960
(*b*) Emptying operation	..	..	2,390	2,240	2,040	1,810	1,690	1,460[c]	1,180	2,130	1,760

[a] Discharge refers to flow into, and out of, lock chamber through ports in river wall. [b] Prototype. [c] 16-ft head. [d] Model.

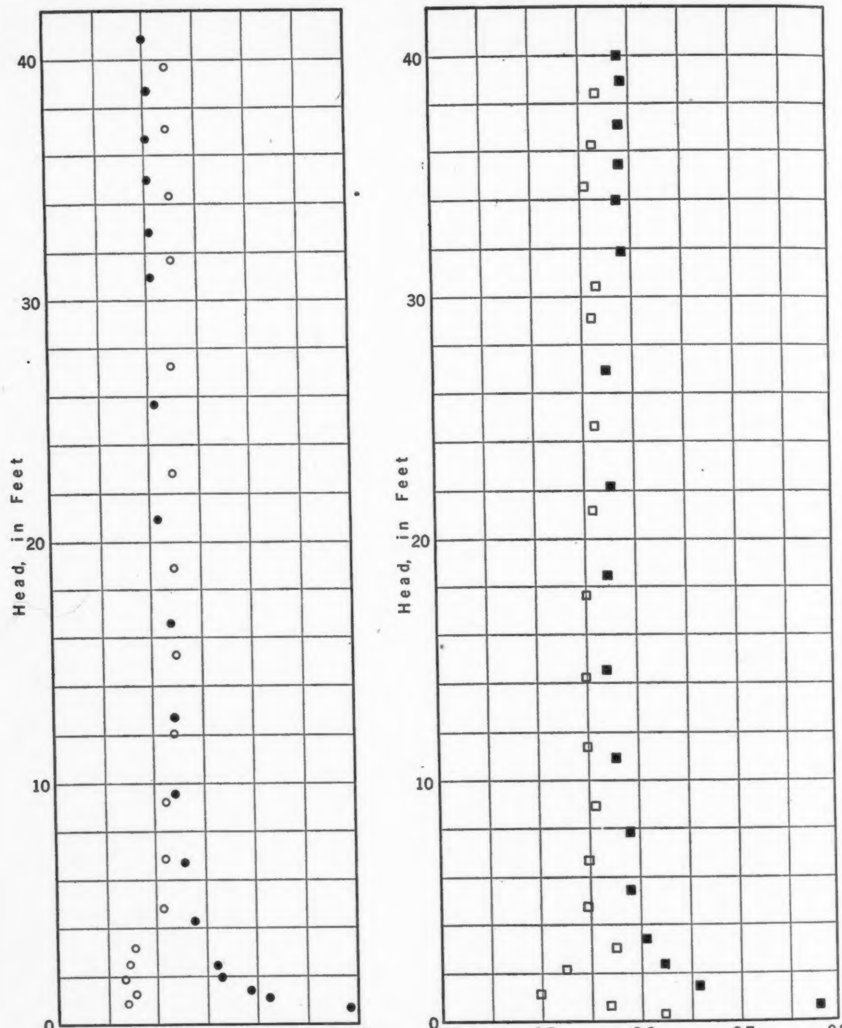
A good agreement is also noted in the distributions of discharge in the lock-chamber ports in emptying operations in one model and prototype locks which are shown in Fig. 30(*b*).

Lock Coefficients.—Filling and emptying coefficients for one of the lock hydraulic system of the prototype and model are shown in Fig. 31. These coefficients are based on observed discharges computed from the average rate-of-rise and rate-of-fall curves and on theoretical discharge computed from stage records (see Eq. 4). The average filling coefficients for heads above 5 ft are 0.61 and 0.62 for the prototype and model, respectively. Corresponding emptying coefficients are 0.58 and 0.56. The lock coefficient varies as the square root of the slope of the rate-of-rise or rate-of-fall curves. For that part of the filling operation for which coefficients were computed, the model had the steeper rate-of-rise curve and the larger filling coefficient, and the prototype had the steeper rate-of-fall curve and the larger emptying coefficient.

As in one lock, the pattern of the coefficients of another indicated that there was less difference between the static and effective heads in the model than in the prototype.

Guntersville Lock—Comparison of Prototype and Model Tests

Intake Ports.—In the model tests on one lock it was found that the distribution of flow through the intake ports was fairly uniform and that it was affected only slightly by variations in the head. The intakes used in one model were also used in another model, and no further tests were made



Description	Legend: ●	○	■	□
Filling operation	Prototype	Model	....	....
Emptying operation	....	....	Prototype	Model
Average coefficient[a]	0.61	0.62	0.58	0.56

[a] For heads greater than 5 ft.

FIG. 31.—LOCK COEFFICIENT

in the latter model to determine the distribution of flow through intake ports. In prototype tests of flow distribution at one lock it was found that the distribution of flow in the four ports on one side of the intake section varied from 20.5% to 28% and that the pattern of flow distribution was similar to that for the intake ports of another lock.

Transition Section.—The prototype and model transition sections are correlated by using the dimensionless coefficient of discharge because corresponding dimensions of the culvert at which pressure measurements were taken were not similar. Coefficients for the prototype and model transition section meter —0.95 and 0.93, respectively—are in good agreement.

Lock Stage Tests.—Stages with respect to time in the lock chambers (see Fig. 21) during filling operations in the model and prototype are shown in Fig. 32. The time required to fill the lock chamber with a 39.2-ft head in the prototype (5 min 30 sec) was 39 sec, or about 11%, less than the time required to fill the lock chamber in the model. The rate-of-rise curves indicate that the water surface rose faster in the prototype than in the model during the time the valves were opening. It is believed that this difference is due largely to a personal factor introduced in the manual operation of the valves in the model. After the valves are open the rate-of-rise curves are in better agreement, having slopes of 2.77 and 2.57 ft per min^2 for prototype and model, respectively. The maximum rates of rise were 12.7 and 12.5 ft per min in the prototype and model, respectively.

Emptying curves are shown in Fig. 33. The same period of time, 7 min 36 sec, was required to empty the model and prototype lock chambers with a head of 39.0 ft, and the emptying curves were practically identical. Maximum rates of fall in the prototype and model were 8.8 and 9.3 ft per min, respectively, and corresponding slopes of the descending rate-of-fall curves were 1.22 and 1.27 ft per min^2.

Lock-Chamber Ports.—The distribution of discharge in the lock-chamber ports observed in filling operations on the prototype, on the Iowa City model, and on a model of the lock tested in the TVA Hydraulic Laboratory at Norris, Tenn., are shown in Fig. 34.

Velocities measured in the lock-chamber ports of the models with steady flow are shown in the distribution of flow curves as percentages of the total flow. Velocities measured in the lock-chamber ports of the prototype lock during filling operations were plotted against time, and from these graphs the port velocity distribution for a given static head was obtained, using the time interval at which the given static head occurred.

The distribution of flow during the filling operation was better in the models than in the prototype. Percentages of total flow carried by the seven ports in the upstream half of the chamber and by the seven ports in the downstream half of the chamber are: For one model (average of four heads), 47% and 53%; for another model (25-ft head), 46.5% and 53.5%; and for the prototype (average of six heads), 40.8% and 59.2%. Patterns of flow distribution in the two models are very similar, and they agree more nearly with that of the prototype at a 35-ft head than with the average prototype distribution after the valves are open.

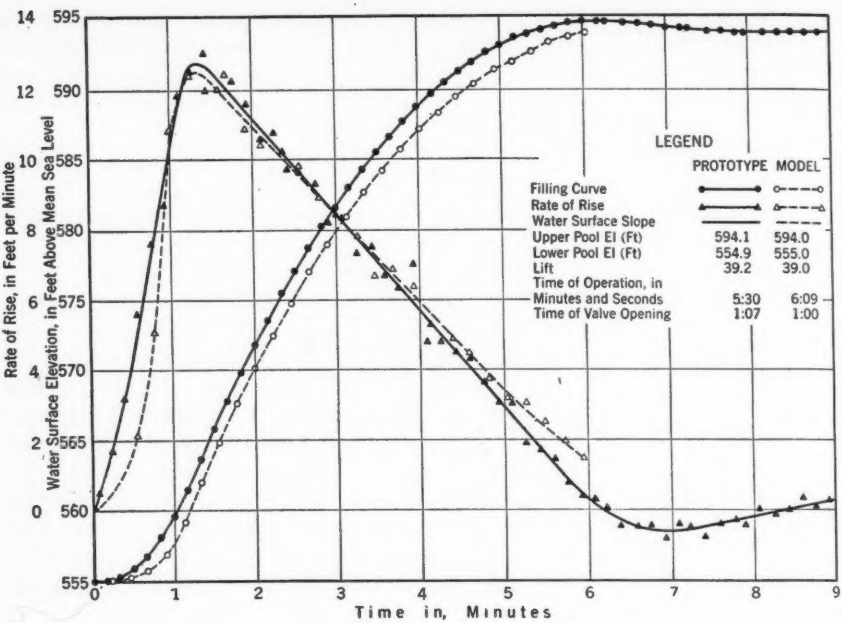


FIG. 32.—FILLING CHARACTERISTICS

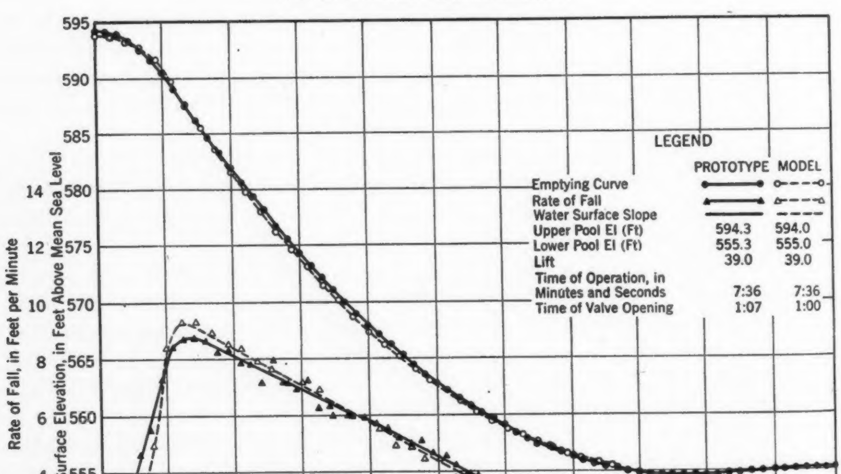
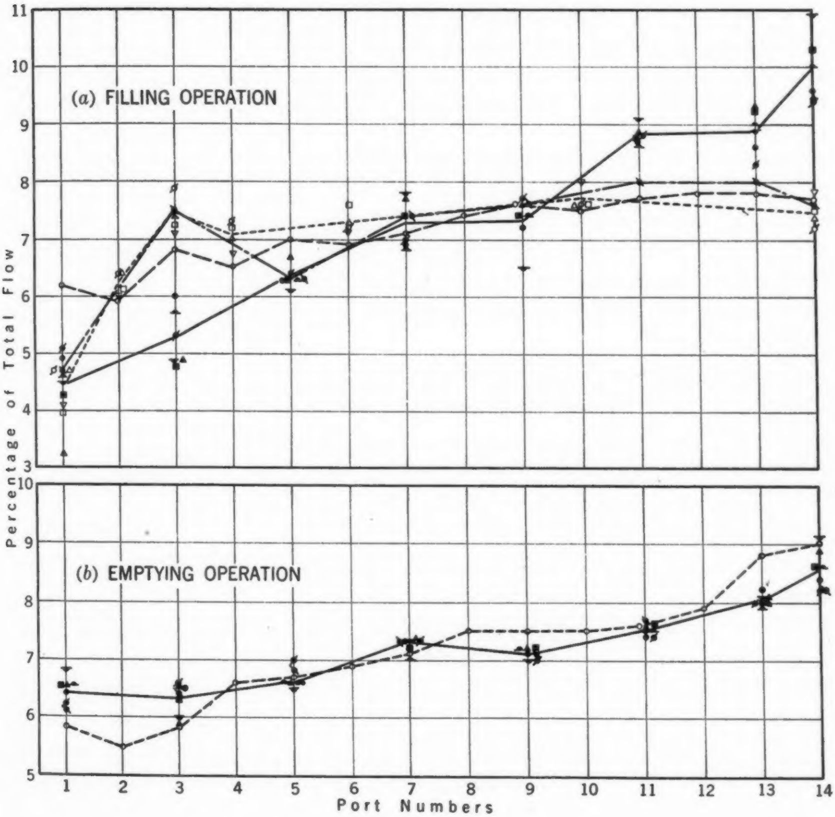


FIG. 33.—EMPTYING CHARACTERISTICS

The distribution of flow in the lock-chamber ports during the emptying operation in the model and prototype was in better agreement than the distribution during filling. Percentages of total flow carried by the seven ports



Description	Average of Six and Seven Heads[c]							One Head[d]	Average of Four Heads[d]			
	⦸	⌀	●	⏶	■	▲	⏷	○	ø	△	□	▽
Head, in feet	35[b]	30	25	20	15	10	5	25	33	25	17	7
Discharge,[a] in Cubic Feet per Second:												
(*a*) Filling operation	..	2,590	2,410	2,180	1,890	1,550	1,120	2,260	2,380	2,060	1,690	1,130
(*b*) Emptying operation	..	1,740	1,600	1,430	1,240	1,020	730	1,640	....	....	....	....

[a] Discharge refers to the flow into the lock chamber through the ports in the land wall. [b] The curve for the 35-ft head is plotted as a separate line in Fig. 34(*a*), the remaining six in this group being averaged. In Fig. 34(*b*), all seven are averaged. [c] Prototype. [d] Model.

FIG. 34.—DISTRIBUTION OF FLOW, LOCK-CHAMBER PORTS (LAND WALL)

in the upstream half, and in the downstream half of the chamber are, for one model (25-ft head), 44.3% and 55.7%, and for the prototype, 46.5% and 53.5%. The models had the more even distribution during the filling operation, and the prototype during the emptying operation.

Discharge Ports.—Only preliminary tests were made on the distribution of flow in the discharge ports of one of the models. The discharge sections of one model were used also in another model, and since the preliminary tests agree with similar data from one of the models, no further tests were considered necessary.

From prototype port velocity tests at one lock, the distribution of flow in the sixteen discharge ports of the land wall were computed for seven heads ranging from 5 to 35 ft. Distribution of flow among the sixteen discharge ports remained approximately the same for the seven different heads, and the culvert flow carried by the eight upstream and eight downstream ports was 43.6% and 56.4%, respectively, based on the average distribution for the seven heads.

Lock Coefficients.—Average coefficients for the lock hydraulic systems in the prototype and model, for heads greater than 5 ft, are:

Observations on:	Filling	Emptying
Prototype	0.95	0.63
Model	0.88	0.63

For the emptying operation the coefficients are in good agreement, having the same average value for prototype and model, but for the filling operation the prototype coefficients average 8% higher than those for the model.

The prototype ports and the model ports for which data are presented in this paper have the same dimensions at the throat, but the prototype ports are larger at the lock-chamber end. Consequently, the divergence angle for the prototype, 11° 44′, is larger than that for the model port, 7° 50′, and the ports are not geometrically similar. This greater divergence angle is undoubtedly a factor contributing toward the higher coefficients and faster rate of filling observed in the prototype.

Although the area of the culverts was used in computing coefficients for the lock hydraulic system, the minimum area in the prototype and in the model is the cumulative area at the throats of the lock-chamber ports. If, instead of the culvert area, the minimum port area was used, the average coefficient for emptying would be 0.78. For emptying one lock the average coefficients based on least port or culvert area were 0.73 and 0.74 for prototype and model, respectively.

The coefficients for a prototype lock based on static head gradually increase as the head decreases, and for the model they gradually decrease as the head decreases, until just before the end of the lock operation where values of the coefficient increase sharply.

Summary and Conclusion

In Table 3 the results of model and prototype studies of three locks are summarized. It includes operation data and areas of critical sections of the lock hydraulic systems.

Tests on one lock offered the best opportunity for comparing model and prototype performance because the corresponding structures were geometrically similar and lock operation data were in agreement. Moreover, it will be noted that this lock had the most consistent agreement between model and prototype of the three locks tested, the only material difference in the performance of the corresponding structures being in the longitudinal slope of the water surface for which readings in the prototype lock were larger than in the model.

For model and prototype locks of another there was agreement in lock operation data, and the two structures were geometrically similar except at the lock end of the lock-chamber ports. A more efficient filling operation was observed in the prototype than in the model, the former having the greater divergence angle in the lock-chamber ports. Furthermore, during the filling operation, the less efficient model had the more even distribution of flow in the lock-chamber ports. It is of interest to note the performance of Guntersville model and prototype locks during the emptying operation. There was agreement in operation time, lock coefficient, and the distribution of flow in the lock-chamber ports irrespective of the difference in lock-chamber port divergence angles. The throat of the lock-chamber port, for which the cumulative area was the minimum in the system, appeared to be effective as the control section of the system during the emptying operation.

There was less agreement in the performance of one model and prototype locks because, in addition to dissimilarity in the throat area of the lock-chamber ports, lift and valve time in the two structures were not in agreement. However, the performance of this model and prototype during the emptying operation, similar to that of other locks, showed agreement in the lock coefficients based on the minimum areas in the respective systems and in the distribution of flow in the lock-chamber ports.

When geometric similarity does not exist in the important elements of two hydraulic structures it is not expected that their operations should conform to the basic laws of similitude. However, where geometric similarity did obtain, and where analogous functions could be correlated by means of the basic laws, this study substantiates the theory upon which the use of hydraulic models is founded and, it is believed, tends to justify the confidence engineers have placed upon this method of analyzing difficult problems in design and operation within the past few years.

Acknowledgments

Col. R. G. Powell, then division engineer, Ohio River Division, Cincinnati, Ohio, sponsored the project and authorized the tests on the prototype locks in December, 1937. The tests were supervised by personnel of the U. S.

Engineer Sub-Office, Iowa City, under the direction of the district engineer, U. S. Engineer Office, St. Paul, Minn.

The prototype lock tests were conducted by Marvin J. Webster, Assoc. M. Am. Soc. C. E., of the Iowa City Sub-Office, with the assistance of personnel from the District Engineer Offices in whose areas the respective locks are located. Some of the material presented in this paper has been drawn from a report on the project prepared by Mr. Webster for which acknowledgment is made. Permission to use the data obtained in the prototype lock tests is acknowledged to Col. J. W. Moreland, M. Am. Soc. C. E., district engineer of St. Paul.

DAM OUTLET WORKS IN A WESTERN STATE

By S. P. Wing,[7] M. Am. Soc. C. E.

Synopsis

The principal hydraulic model tests of the dam outlet works (34), which included flow through the intake tower, penstock turbine, and outlet-valve manifolds, were made on a $5\frac{5}{8}$-in. (1 : 64 scale), extremely smooth, sheetmetal model which had a friction coefficient of $f = 0.021$ at a Reynolds number **R** of 80,000 and used a maximum of 2 cu ft per sec of water. Performance and efficiency tests of the 30-ft diameter prototype, utilizing a maximum discharge of 20,000 cu ft per sec with measured velocities up to 90 ft per sec and at a Reynolds number of from 40 to 70×10^6, made it possible to obtain data from which various hydraulic features of the model and prototype could be compared. These included loss coefficients for the trashrack, bends, branches, valves, and straight-pipe friction. A full discussion of the data was presented before the hydraulic section at the Annual Convention of the Society in July, 1940 (35). This paper is a summary of such of the results as may be generally useful in hydraulic design, together with suggestions concerning the technique of field measurements which, if found practicable, may make possible better comparisons between models and prototypes in future tests.

The letters in Figs. 35 and 36 indicate points at which measurements were taken. Distances are given in units of pipe diameters, that of the 25-ft section being converted into its 30-ft hydraulic equivalent. Three series of comprehensive tests were run from 1937 to 1939 with heads varying from 213 to 337 ft. During the tests the turbines were shut down, varying discharges being obtained by manipulating the 84-in. outlet valves. Pressures were measured with calibrated Bourdon gages and differential mercury manometers, utilizing the dead-ended penstocks for piezometric connections. Tests showed that such large connections taking off at an angle resulted in the gages registering an excess pressure of $0.04\ h_v$.

Discharges were obtained by impact-pressure tube traverses at right angles to one another in a 96-in. section of pipe located immediately downstream from a reducer and in front of valve A_1, point K. Utilizing these measurements and the simultaneously measured heads on the other valves, the total discharge for the entire battery of valves was computed. The coefficient of the pitot tube in the formula,

$$Q = \frac{A}{C} \sqrt{2\,g\,H} \qquad (6)$$

[7] Civ. Engr., Bureau of Reclamation, Denver, Colo.

obtained from a still-water rating, was corrected for wall proximity effects and for the effects due to curved streamlines caused by the valve and the obstruction of the streamlined struts used to support the instrument (see Fig. 37). These latter corrections were determined by the photoelastic labora-

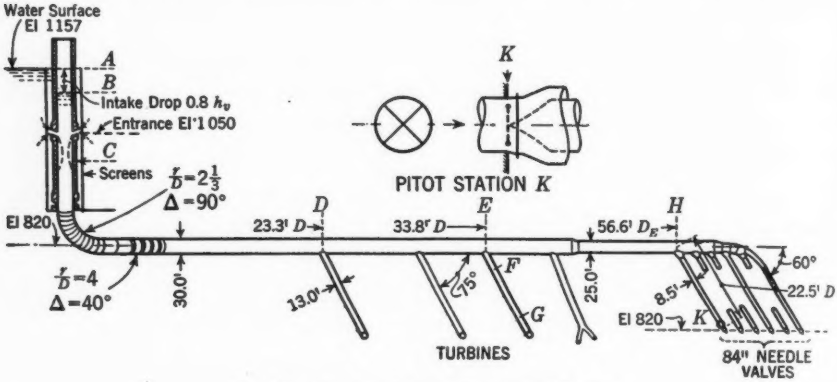


FIG. 35.—SCHEMATIC VIEW OF OUTLET WORKS

tory of the Bureau of Reclamation, using the electric-analogy method (see Fig. 38). Fig. 39 shows the results found by the laboratory, the close correspondence between the laboratory result and that obtained in the rating channel for 2% obstruction being of interest. Since the obstruction offered

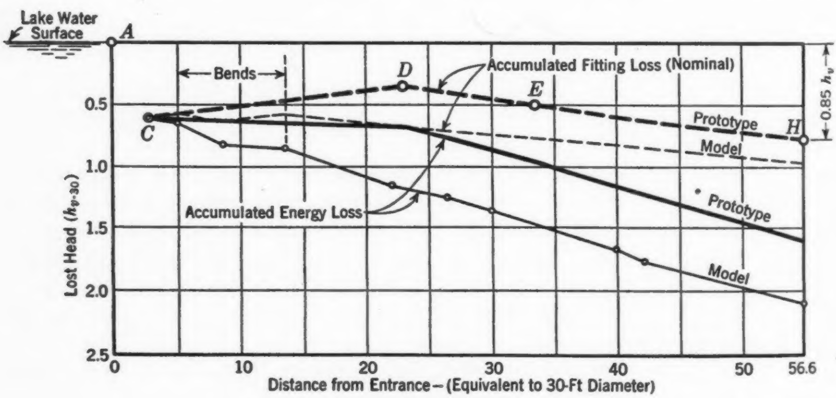


FIG. 36.—ENERGY LOSSES

by the struts is a variable percentage in each circular lamina of the pipe, separate coefficients were applied to each individual reading of the pitot traverse.

Plots of the velocity and pressure distribution in the cross section indicated that upstream from the pitot tube the friction in 22.5 diameters of pipe plus a 15% reducer did not introduce enough resistance to normalize the upset

velocity distribution due to branching, the velocity contours showing the crescent-like form (typical of flow below a bend) superimposed on the upstream cone of pressure caused by the downstream valve. The latter caused differ-

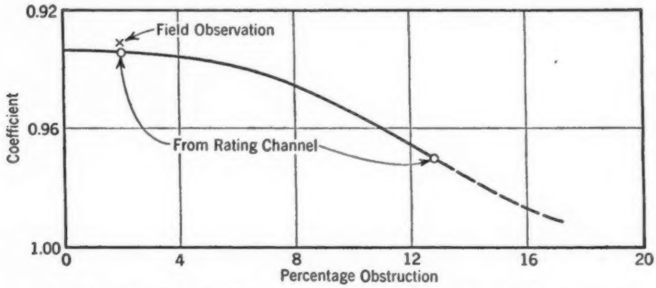


FIG. 37.—PITOT-TUBE COEFFICIENT VERSUS PERCENTAGE OBSTRUCTION

ences in pressure up to 40 ft of water, or nearly 0.5 h_v, between the pipe wall and the center of the pipe. Worth considering in valve design is the fact that the energy of flow was also unsymmetrical, the half of the pipe away from the center of curvature of the branch carrying 10% to 15% h_v more energy than the inside, with the possibility that pressure unbalance existed within the valve.

On the whole, the pitot-tube gagings were reasonably self-consistent. An estimate of their errors and a comparison with current-meter gagings in the river indicated a standard error for the largest discharges of the order of 2%.

The interpretation of losses in a penstock, with fittings but relatively few diameters apart, depends upon an ability to separate fitting losses from those due to straight pipe. A pipe entrance or fitting tends to result in a contracted jet of high-velocity water which changes in 20 to 200 diameters downstream to a normal velocity distribution, the length required being dependent on the roughness of the pipe wall and on Reynolds' number **R**. If knowledge of the energy grade is based only on the readings of wall piezometers, the profile of the energy grade characteristically will be found to show an apparent increase in energy in the first 3 or 4 diameters following a fitting, pass through a point of inflection and then will become concave upward, approaching asymptotically the friction gradient. The model test, moreover, indicated the possibility that nodes of pressure of about 0.03 h_v, located 3 to 4 diameters apart possibly

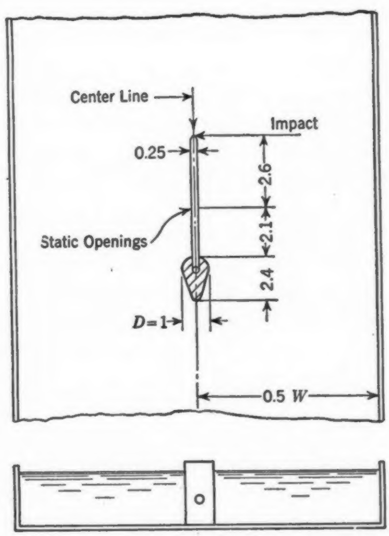


FIG. 38.—TRAY HOLDING ELECTROLYTE

caused by the expansion and contraction of the high-velocity jet, were superimposed on the profile immediately downstream from the branch fitting. This possibility requires more investigation before it can be taken as a fact.

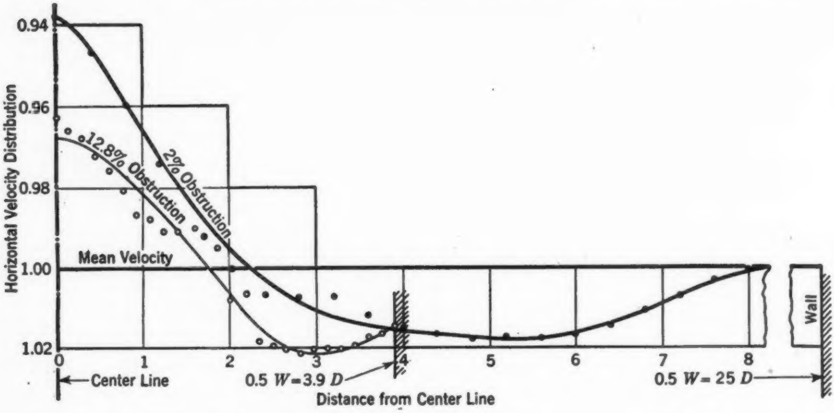


FIG. 39.—VELOCITY DISTRIBUTION VERSUS WIDTH IN THE PLANE OF THE STATIC OPENINGS

The foregoing phenomena make it almost impossible in a power installation to find a run of pipe in which it can be certain that only straight-pipe friction is reflected in differences in the readings of wall piezometers. For the present experiments, the most reliable result estimates of straight-pipe friction were obtained by utilizing the measurements made in the 13-ft turbine penstocks in connection with the Gibson (36) efficiency tests of the turbines. The measuring section F G of Fig. 35 included 8 diameters of pipe, located 8 diameters from the branch. The results, Fig. 40, at $\mathbf{R} = 20 \times 10^6$, gave a value of $f = 0.0207$. Utilizing the model tests, which showed that in such a location about 0.03 h_v of the measured drop should be ascribed to the upstream diversion loss, the friction coefficient for 13-ft diameter straight pipe was estimated at $f = 0.0177$, corresponding to Manning's $n = 0.015$, to Scobey's $K_s = 0.475$, and to Nikuradse's sand roughness factor $\frac{K}{\mathbf{R}} = 0.00134$. This latter factor can be interpreted as meaning that the bitumastic-covered surface of the 13-ft pipe was as rough hydraulically as if it had been coated with uniform sand grains 0.1 in. in diameter. According to the field observers, broom marks and joints were readily visible in the bitumen, a 20-ft straightedge showed $\frac{1}{8}$-in. openings below it, and flattened bolt heads projected $\frac{1}{8}$ in. from the welded interior of the pipe. Friction coefficients for other sizes of pipe were computed from these measurements, assuming them to vary inversely as $D^{1/3}$, the value for the 30-ft pipe being $f = 0.0134$. These unexpectedly high values can be compared with the results obtained from concrete pipe at comparable values of $\mathbf{R}$ plotted on the graph of Fig. 40. Also on the same graph are plotted overall losses per diameter for each of the sections measured—A D, A E, A H, etc. (see Fig. 35). Since the values are approximately constant at 0.03 h_v per

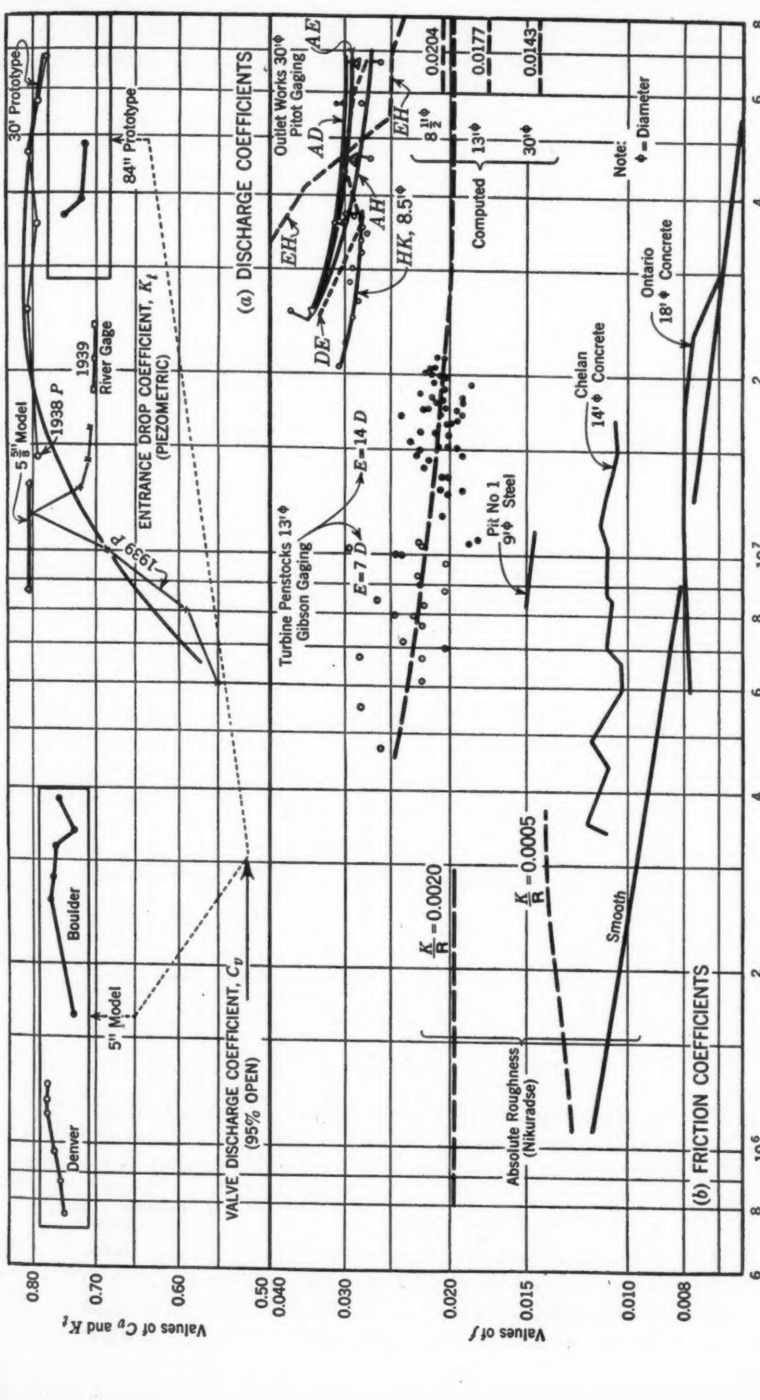


FIG. 40.—COMPARISON OF FRICTION FACTORS

diameter, this figure can be used as an over-all check for hydraulic designs in which the hydraulic losses due to friction and fittings are separately estimated.

The general results of the tests are best shown in the form of graphs. Fig. 36 shows, in terms of velocity head, profiles of the accumulated total energy loss and of the total hydraulic loss ascribable to fittings for both the model and prototype. Fig. 40(*a*) is a comparison of the drop in the intake tower and the valve discharge coefficients plotted against **R**. For the former a smooth curve has been passed through the 1939 and 1938 measurements, based

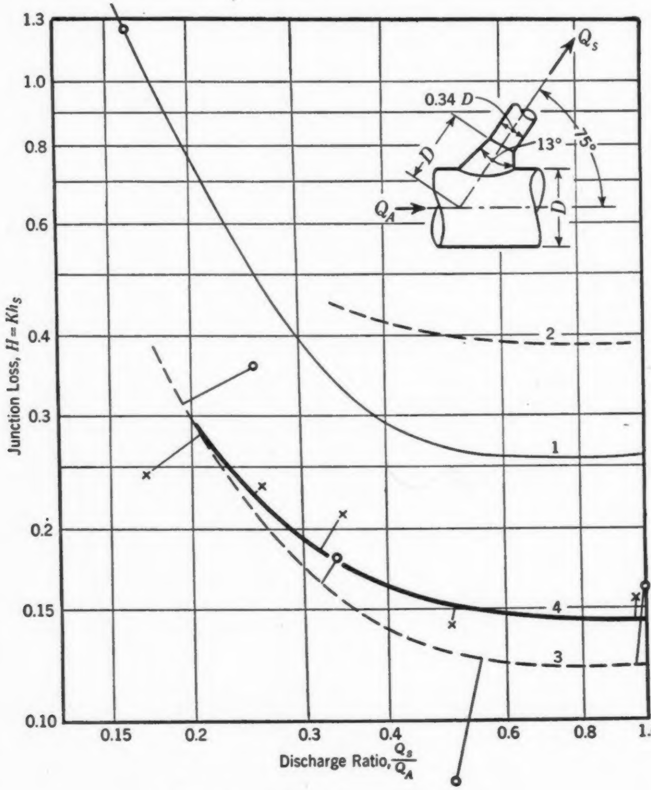


FIG. 41.—JUNCTION LOSS COEFFICIENTS

on velocities as determined by the pitot tube. The results from the river gagings are shown for comparison. Fig. 41 shows branch losses at the valve manifold plotted against the percentage of total flow diverted to the branch:

Curve (Fig. 41)	Structure	Reynolds' number (in millions)
1	Bureau of Reclamation model	0.05
2	Thoma model (interpolated values)	0.10
3	Commercial laboratory	0.30
4	Bureau of Reclamation prototype	40.00

Table 4 compares the various loss coefficients for the entire structure as originally estimated, as found from the model, and as estimated from the prototype tests.

Quantitative correspondence between model and prototype can be expected only where Reynolds' criteria can be satisfied and where absolute roughness in the model can be scaled down in accordance with model laws. This was not possible in the Boulder model which, while as smooth as possible, was materially rougher than demanded by similitude. Referring to the fitting loss of Fig. 36, it can be seen that in the prototype at point D, 23 diameters from the entrance, there was an indicated loss of about one half that measured in the model and at point H, 57 diameters downstream, a value 85% as large. It will be noted, moreover, that at point D the accumulated fitting loss in the prototype is shown as only one half that observed 20 diameters upstream immediately downstream from the intake. It will be realized at once that this decrease in accumulated loss downstream as measured by wall piezometers is due to the fact that the profiles are only nominal energy grades and include no corrections for actual velocity distributions. It appears probable from these tests that the prototype, with relatively less friction than the model, requires a greater length, measured in diameters, to regain its pressure head following the contraction below the intake. The required length may be analogous to the distance required to produce normal distribution below an intake in a smooth pipe, for which S. Goldstein (37) gives the formula

$$L = 0.7\, D\, \mathbf{R}^{0.25} \qquad (7)$$

Applied to the present tests, four times the length required in the model, measured in diameters, would be required to produce normal velocity distribution in the prototype. Whatever the reason, it is believed that the over-all measured prototype fitting loss, 0.85 h_v (which includes entrance, two bends,

TABLE 4.—COMPARISON OF LOSS COEFFICIENTS

No.	Description	Loss Coefficient in Terms of Velocity Head of 30-Ft Diameter Pipe		
		Preliminary design	Model	Prototype
1	Screens, h_v of 30-ft pipe	....	0.10	0.01
2	Intake drop from the lake to within the tower	....	0.81	0.76 ±
3	Estimated entrance loss 57 diameters from intake	1.10	0.68	0.63
4	Estimated vertical and horizontal bend loss	0.40	0.11	0.08
5	Estimated losses for eight 12-in. tie rods and four 13-ft branch openings	0.20	0.17	0.12
6	Estimated 30 by 25 reducer loss	0.05	0.04	0.02
7	Measured total entrance and fitting losses at 57 diameters	1.75	1.00	0.85
8	Branch loss with 100% diversion, measured 30 diameters from branch	....	0.26	0.14
9	Valve coefficient	0.72	0.76	0.72
10	Friction coefficient f for 30-ft pipe	0.012	0.021	0.0134

the disturbances caused by openings for four branches, together with their eight tie rods, and a 30 by 25 reducer), provides a satisfactory check on the model results. In Table 4 this over-all loss has been broken down into its

components and compared with the standard loss coefficient originally used for the preliminary design of the outlet works and with those later obtained from the model. Although the breakdowns are only estimates based on a consideration of all the data, these fitting coefficients should be helpful to designers until better values become available. It is estimated that the values have a standard error of the order of 15%.

The entrance loss into the tower through its radial ports, 0.63 h_v as given from the foregoing breakdown, is sometimes confused with the piezometric drop which occurs from the lake to the water surface within the tower. The latter (as is shown in the bulletin (34) on the model tests) involves a reaction head due to the water turning downward within the tower and is not an absolute measure of entrance losses. This coefficient of tower drop, however, as obtained from various gagings, forms a good means of comparing performance of model and prototype. The results are shown in Fig. 40. Although at first sight the variations in the different tests may seem large, the 12% range in the experimental values is believed comparable to the precision of the majority of prototype tests. The increase in the coefficient with increasing **R**, as indicated by the curve averaging the 1939 and 1938 prototype results, may, perhaps, be partly accounted for by the formation of vortices at the intake which were observed in the 1938 measurements. At that time the head on the intake was only from four to six times the mean velocity head. Since the velocity heads for some of the stream filaments passing around certain unavoidable sharp edges of the intake, measured in terms of the mean velocity, may have been even a larger multiple of the mean head with the corresponding absolute pressures approaching zero, this is a condition which should be taken into account by designers, as the vacuum may cause cavitation and the entraining by vortices of large quantities of air. Without complete exploration of pressures in a model, it is a condition that is difficult to predict.

The model showed a loss through the screens of 0.10 h_v. In the prototype the loss was less than one tenth of this value. This undoubtedly was due to the fact that velocity through the model screens was below Reynolds' critical value. A model should not reproduce from the prototype details within which subcritical velocities may exist during the model tests.

Of considerable interest to designers is the loss due to a closely connected 90° vertical and 40° horizontal bend. Although the computed loss coefficient for the prototype of 0.08 h_v may have a considerable percentage error, the total measured loss for all the fittings was such that the bend loss, regardless of assumption, must have been small. To the writer's knowledge this is one of the few prototype tests that has been made on bends.

The junction loss for the branch from the manifold is shown in Fig. 41. Curve 1 for the model is interpolated from a very complete series of tests described in the bulletin (34). Curve 2 was likewise interpolated from D. Thoma's well-known tests (38). Curve 3 was obtained from the results of a commercial laboratory in connection with the design of the turbine manifold. It will be noted that the observations defining its averaging curve are widely scattered. Any one who has examined critically the technique by which fitting losses must be obtained, as set forth in the bulletin (34), will

appreciate how difficult it is to obtain reliable results and will know that only in the exceptional test can the absolute values of the results be accepted. It is enough to state that coefficients of fitting loss are not fixed quantities depending merely on shape. They tend to decrease with increasing smoothness and with increase of Reynolds' number, and to be greater the greater the distance below the fitting at which they are measured. Also, it is to be noted that with multiple similar branches from a manifold, the loss will be different at each successive offtake due to change in the value of the percentage of the main flow diverted.

The needle valve discharge coefficients are shown in Fig. 40. It will be noted that in the model with increasing **R** the coefficient increased, whereas in the prototype it decreased. The first case undoubtedly reflects the decrease in friction through the valve with increase in **R**. In the second case an analysis of the velocity distribution due to a curved streamline within the valve indicated the probability that at certain points vacuum was approached at high values of **R**. Once this point was reached, the effective area of the waterway would be reduced, cavitation would commence, and the discharge coefficient would decrease. The performance of the prototype was in line with these predictions.

It is believed that on the whole the use of a model for predicting quantitative results in the outlet works was pre-eminently successful, nearly halving the error in design that would have existed without its use. On the other hand, much more experimental data and development of theory are needed if predictions of losses within 10% are desired in the prototype. Particularly required are methods for forecasting pipe friction in terms of a quantitative pipe roughness and for estimating fitting losses at various distances downstream in the stretch before velocity distribution has become normal. Finally, it must not be forgotten that there is no way in which a factor of safety can be applied in the economic layout of the flow line of a hydroelectric development. An overestimate of hydraulic losses leads to the overdimensioning of the conduit with excess interest charges in perpetuity, whereas an underestimate leads to continuous waste of power in excess friction losses.

Acknowledgment

The field work was directed by John Parmekian and N. G. Noonan. Especial appreciation is due R. E. Kennedy, M. Am. Soc. C. E., who made the computations.

The hydraulic laboratory of the Bureau of Reclamation is under the direct charge of Jacob E. Warnock, M. Am. Soc. C. E. All design and research work is under J. L. Savage, Hon. M. Am. Soc. C. E., chief designing engineer. S. O. Harper, M. Am. Soc. C. E., is chief engineer, and the Bureau is directed by John C. Page, M. Am. Soc. C. E., commissioner.

SPILLWAY COEFFICIENTS AT AN EASTERN DAM

By G. H. Hickox,[8] M. Am. Soc. C. E.

Synopsis

The determination of discharge coefficients for the dam spillway crest gates is described in this paper. Actual coefficients calculated for the gates from records of their performance during the flood of January and February, 1937, are compared with the coefficients predicted from tests made on a 1 : 72 scale model by the Bureau of Reclamation. This comparison indicates that the coefficients predicted by the model average 3.9% larger than those actually measured on the dam during this period.

General Conditions

The dam was built as a part of a program of navigation, flood control, and power development. It controls the discharge of 2,912 sq miles of land drained by two rivers.

It is a concrete gravity structure having a total length of 1,860 ft and a maximum height, to the roadway on the top at El. 1061, of 265 ft. The spillway crest is at El. 1020 and is 300 ft long, divided into three bays each 100 ft long. Three hydraulically operated steel drum gates are installed on the concrete spillway crest, and the crests of the gates may be raised to any elevation between 1020 and 1034 to retain flood peaks. Eight outlet conduits permit the discharge of water when the reservoir level is below the spillway crest. The maximum discharge of these conduits with reservoir level at El. 1020 is approximately 37,000 cu ft per sec. The power house contains two generating units, each having a capacity of 56,000 kva.

Fig. 42 shows a general plan and elevation of the dam and a section through the spillway. Fig. 43 gives the dimensions of the spillway crest in detail.

Flood of January, 1937.—Unusually heavy rains resulted in flood stages. During this period the entire flow of rivers above the dam was stored in a lake. This necessitated raising the spillway drum gates to hold water a maximum of about 11 ft above the concrete spillway crest. Late in January it was seen that moderate discharges at the dam would not cause further rises on the river, and accordingly water was released from the lake to provide storage space for additional runoff. Much of the water was released by lowering the crest gates and allowing it to discharge over the spillway. Advantage was taken of this opportunity to obtain the necessary measurements for determining the discharge coefficients of the spillway for heads on the gates as great as 9.8 ft and for discharges as great as 30,000 cu ft per sec.

[8] (Deleted by censors.)

Data Available

Gate Operating Records.—The program of water release at the dam required the discharge to be held approximately constant for considerable

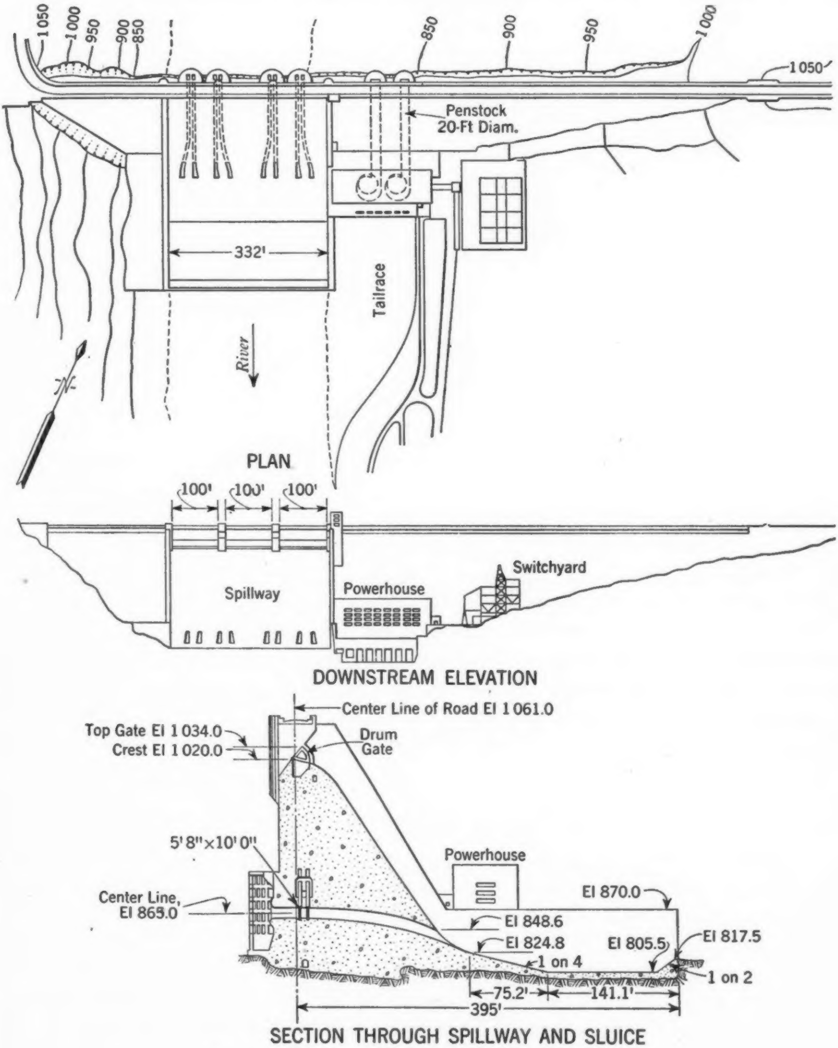


Fig. 42.—Plan, Elevation, and Section of Dam

periods of time. It was necessary to change the elevation of the crest gates, since the water level in the reservoir was changing. Accordingly, the gates were reset about every two hours. A complete record of the operation of the spillway crest gates, giving the time at which changes were made in the gate

settings and the elevations to which the gates were set, was available from the records kept at the power house. Records of reservoir water-surface elevations were also available.

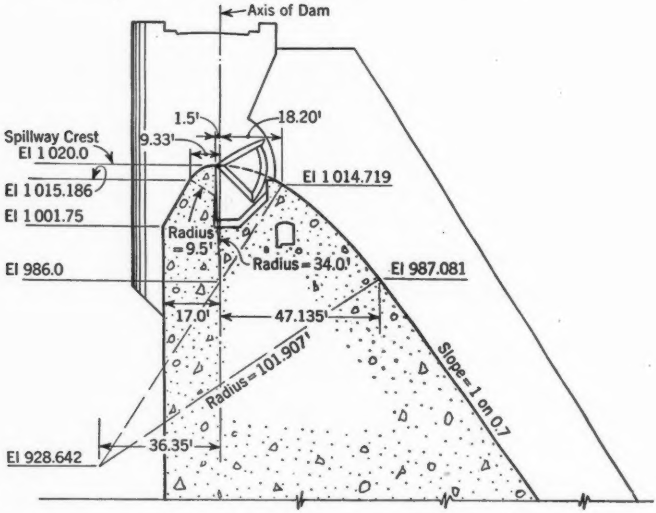


FIG. 43.—DIMENSIONS OF SPILLWAY CREST

Discharge Records.—A gaging station has been established by the U. S. Geological Survey at a point 0.9 mile below the dam. A standard recorder was installed, and a continuous record for the period of spillway discharge was available. The rating curve is well defined since the control is a solid rock diagonal reef about 150 ft below the gage. A number of discharge measurements were made during the flood period. Only one of these is as much as 4% from a mean rating curve, and most of the measurements are less than 1% from the curve.

CALCULATION OF DISCHARGE COEFFICIENTS

Notation.—The letter symbols used in this paper are defined where they first appear and are assembled for convenience of reference under "Bibliography and Notation" at the end of the Symposium.

Correlating the Data.—In computing the discharge coefficients, it was necessary that the discharge over the spillway should be the same as that at the gaging station and that the river stage at the gaging station should be steady. As previously mentioned, the gates were adjusted about every two hours, causing fluctuations in the discharge. In most cases a 2-hr period was sufficient to allow the stage at the gage to become steady, since the changes in discharge were usually small. When observations were used which were only two hours apart, the elevations of the gate crest and the reservoir level were taken as of immediately before the change—that is, at the end of the 2-hr period. The gage height at the gaging station was also taken at the end of the period.

When large changes in discharge occurred, a greater time was required for the river stage to become steady. When it appeared from the recorder chart that a steady condition had not been established at the gage, the data were not used. No data were used to determine discharge coefficients if there was any doubt as to their value, either because of unsteady flow or for any other reason. Use of river discharge at the gaging station to represent the discharge over the spillway at the same time is justified because of the care in rejecting data which did not represent steady flow and because of the relatively small storage between the dam and the gaging station.

Table 5, Cols. 1 to 8, inclusive, lists the data regarding reservoir and gate-crest elevations obtained from the power-house records. Col. 9, Table 5, gives the gage height, in feet, at the gaging station, corrected from the continuous chart.

Calculation of Discharge.—A rating curve for the gaging station was constructed from the discharge measurements by plotting the discharges against gage heights and drawing a curve through them so as to give greatest weight to those measurements that were made during the period of observation.

There is practically no inflow to the river between the dam and the gaging station. The gaging station records indicate that, just before spillway discharge began, the combined inflow and leakage through the dam was about 50 cu ft per sec. Col. 10, Table 5, gives discharges taken from the rating curve, less 50 cu ft per sec for all discharges less than 10,000 cu ft per sec. For discharges greater than 10,000 cu ft per sec, the correction was neglected since it amounted to less than 0.5%. The discharges of Col. 10, determined as described, are considered to be the spillway discharges for the corresponding reservoir and gate-crest elevations of Cols. 3 to 7.

Calculation of Discharge Coefficient C.—The discharge coefficient, C, was computed by means of the customary weir equation,

$$Q = C\, b\, H^{1.5} \quad (8)$$

in which: Q = discharge, in cubic feet per second; C = discharge coefficient; b = length of weir crest, in feet (for one dam, 300 ft); and H = head on weir crest, in feet.

Cols. 4, 5, and 6, Table 5, show that the crest gates were not all at the same elevation but varied somewhat, giving different heads on the gates. The discharge coefficient varies slightly with both the elevation of the gate crest and the head, but the variation is not perceptible for the differences observed. The maximum variation of head between gates is 5.7% for Observation No. 1. The results of the first six observations are quite widely scattered, and, if these are omitted from consideration, the maximum variation of head is 1.5% in Observation No. 7. Variation of the discharge coefficient due to this change in head may be neglected. A slight error is also introduced if the average head is used in the weir formula, but with a maximum head difference of 1.5%, this error also may be safely neglected. Col. 8, Table 5, gives the average head on the three gates for each observation. Values of C calculated from

TABLE 5.—DISCHARGE COEF-
(Calculation of Prototype Coefficients

Observation No.	Time	Reservoir elevation[a]	Elevation[a] of Crest			Average elevation[a] of crest
			Gate 1	Gate 2	Gate 3	
(1)	(2)	(3)	(4)	(5)	(6)	(7)
	January 31					
1	11:05 a.m.	30.35	29.28	29.28	29.34	29.30
2	1:10 p.m.	30.41	28.68	28.68	28.74	28.70
3	3:10 p.m.	30.45	28.27	28.27	28.33	28.29
4	5:00 p.m.	30.47	27.86	27.86	27.93	27.88
	February 1					
5	8:30 a.m.	30.70	27.56	27.56	27.63	27.58
6	11:05 a.m.	30.73	27.05	27.05	27.12	27.07
7	7:00 p.m.	30.78	26.10	26.10	26.17	26.12
8	9:00 p.m.	30.78	25.74	25.74	25.82	25.77
9	11:00 p.m.	30.78	25.54	25.54	25.62	25.57
	February 2					
10	1:00 a.m.	30.77	25.20	25.20	25.27	25.22
11	3:00 a.m.	30.77	24.90	24.90	24.97	24.92
12	5:00 a.m.	30.75	24.55	24.55	24.62	24.57
13	7:15 p.m.	30.63	24.55	24.55	24.62	24.57
	February 3					
14	8:50 a.m.	30.49	24.44	24.44	24.52	24.47
15	5:00 p.m.	30.37	24.34	24.34	24.42	24.37
16	10:15 p.m.	30.32	24.24	24.24	24.32	24.27
	February 4					
17	8:25 a.m.	30.17	24.04	24.04	24.12	24.07
18	12:45 p.m.	30.08	23.74	23.74	23.82	23.77
19	1:45 p.m.	30.03	23.54	23.54	23.62	23.57
20	2:45 p.m.	30.01	23.13	23.13	23.21	23.16
21	4:45 p.m.	29.98	22.53	22.53	22.61	22.56
22	8:45 p.m.	29.86	22.33	22.33	22.41	22.36
	February 5					
23	2:40 a.m.	29.73	22.12	22.12	22.21	22.15
24	10:00 a.m.	29.54	21.92	21.92	22.00	21.95
25	1:00 p.m.	29.45	21.82	21.82	21.90	21.85
26	2:00 p.m.	29.49	21.72	21.72	21.80	21.75
27	11:45 p.m.	29.13	21.52	21.52	21.60	21.55
	February 6					
28	3:15 a.m.	29.03	21.32	21.32	21.40	21.35
29	12:00 m.	28.77	21.12	21.12	21.20	21.15
30	2:00 p.m.	28.68	21.02	21.02	21.10	21.05
31	4:30 p.m.	28.61	20.00	20.00	20.00	20.00
	February 7					
32	12:05 a.m.	28.37	20.71	20.71	20.79	20.74
33	1:30 a.m.	28.33	20.61	20.61	20.69	20.64
34	4:30 a.m.	28.23	20.51	20.51	20.60	20.54
35	8:00 a.m.	28.16	20.41	20.41	20.50	20.44
36	10:00 a.m.	28.17	20.00	....	20.00	20.00
37	12:00 m.	28.13	....	....	20.00	20.00
38	2:05 p.m.	28.10	....	20.11	....	20.11
39	4:10 p.m.	28.07	20.11	....	....	20.11
40	7:25 p.m.	28.05	20.31	20.31	20.40	20.34
41	10:10 p.m.	27.95	20.21	20.21	20.30	20.24
	February 11					
42	6:00 a.m.	31.05	22.63	22.63	22.71	22.66
43	9:00 a.m.	31.09	22.28	22.28	22.36	22.31
44	12:00 m.	31.10	21.72	21.72	21.80	21.75
45	12:00 mid.	30.96	21.12	21.12	21.20	21.15
	February 12					
46	4:30 a.m.	30.86	21.02	21.02	21.10	21.05
47	8:00 a.m.	30.74	20.91	20.91	21.00	20.94
48	10:00 a.m.	30.67	20.81	20.81	20.89	20.84
49	2:00 p.m.	30.58	20.71	20.71	20.79	20.74
50	7:00 p.m.	30.43	20.61	20.61	20.69	20.64
51	9:30 p.m.	30.35	20.51	20.51	20.60	20.54
	February 13					
52	2:30 a.m.	30.16	20.31	20.31	20.40	20.34
53	5:00 a.m.	30.07	20.21	20.21	20.30	20.24
54	10:50 a.m.	29.82	20.11	20.11	20.20	20.14
55	1:45 p.m.	29.72	20.00	20.00	20.00	20.00
56	2:15 p.m.	29.70	20.00	20.00	20.00	20.00
57	3:15 p.m.	29.65	20.00	20.00	20.00	20.00

[a] Add 1,000 to Cols. 3 to 7, inclusive, to obtain actual elevations.

FICIENTS AT SPILLWAY
and Comparison with Model Tests)

Average head, *H*, in ft (8)	Gage height, in ft (9)	Corrected discharge, *Q*, in cu ft per sec (10)	C (11)	K (12)	n (13)	C_m (14)	% difference (15)	Observation No. (1)
1.05	2.34	1,150	3.56	3.82	1.506	3.82	+ 7.3	1
1.71	3.35	2,800	4.17	3.77	1.510	3.81	− 8.6	2
2.16	3.87	3,760	3.96	3.74	1.512	3.78	− 4.5	3
2.59	4.56	5,200	4.16	3.70	1.516	3.77	− 9.4	4
3.12	4.71	5,570	3.37	3.66	1.519	3.73	+10.7	5
3.66	5.29	7,050	3.36	3.59	1.523	3.70	+10.1	6
4.66	6.41	10,300	3.40	3.44	1.533	3.61	+ 6.2	7
5.01	6.80	11,500	3.43	3.37	1.538	3.57	+ 4.1	8
5.21	7.00	12,100	3.39	3.33	1.541	3.56	+ 5.0	9
5.55	7.36	13,200	3.36	3.27	1.545	3.53	+ 5.1	10
5.85	7.77	14,500	3.41	3.21	1.549	3.50	+ 2.6	11
6.18	8.06	15,300	3.31	3.13	1.555	3.47	+ 4.8	12
6.06	7.69	14,200	3.18	3.13	1.555	3.44	+ 8.2	13
6.02	8.03	15,200	3.43	3.11	1.557	3.45	+ 0.6	14
6.00	7.74	14,400	3.27	3.08	1.559	3.42	+ 4.6	15
6.05	7.65	14,100	3.16	3.06	1.561	3.43	+ 8.5	16
6.10	7.55	13,800	3.05	3.02	1.565	3.38	+10.8	17
6.31	7.87	14,800	3.12	2.95	1.571	3.36	+ 7.7	18
6.46	8.11	15,500	3.15	2.91	1.575	3.35	+ 6.4	19
6.85	8.55	16,800	3.13	2.84	1.582	3.32	+ 6.1	20
7.42	9.12	18,500	3.06	2.75	1.591	3.30	+ 7.8	21
7.50	9.54	19,800	3.22	2.73	1.593	3.30	+ 2.5	22
7.58	9.53	19,800	3.16	2.71	1.596	3.28	+ 3.8	23
7.59	9.46	19,600	3.13	2.69	1.598	3.28	+ 4.8	24
7.60	9.52	19,700	3.13	2.68	1.599	3.27	+ 4.5	25
7.74	9.59	20,000	3.10	2.67	1.600	3.28	+ 5.8	26
7.58	9.56	19,900	3.18	2.66	1.602	3.27	+ 2.8	27
7.68	9.52	19,700	3.08	2.64	1.604	3.27	+ 6.2	28
7.62	9.66	20,200	3.21	2.62	1.607	3.25	+ 1.2	29
7.63	9.69	20,300	3.21	2.62	1.608	3.28	+ 2.2	30
8.61	10.95	24,100	3.18	2.57	1.617	3.32	+ 4.4	31
7.63	9.68	20,200	3.19	2.60	1.611	3.25	+ 1.9	32
7.69	9.70	20,300	3.18	2.60	1.612	3.28	+ 3.1	33
7.69	9.73	20,400	3.20	2.59	1.613	3.26	+ 1.9	34
7.72	9.73	20,400	3.18	2.59	1.613	3.26	+ 2.5	35
8.17	7.93	14,900	3.19	2.57	1.617	3.29	+ 3.1	36
8.13	5.23	6,880	2.96	2.57	1.617	3.29	+11.2	37
7.99	5.23	6,880	3.04	2.57	1.617	3.29	+ 8.2	38
7.96	5.16	6,670	2.97	2.57	1.617	3.26	+ 9.8	39
7.71	9.58	19,900	3.10	2.58	1.614	3.25	+ 4.8	40
7.71	9.76	20,500	3.19	2.58	1.614	3.25	+ 1.9	41
8.39	10.75	23,500	3.23	2.75	1.589	3.33	+ 3.1	42
8.78	11.19	24,800	3.18	2.72	1.592	3.32	+ 4.4	43
9.35	12.10	27,500	3.21	2.67	1.600	3.34	+ 4.1	44
9.81	13.02	30,300	3.29	2.63	1.607	3.37	+ 2.4	45
9.81	12.95	30,100	3.27	2.62	1.607	3.35	+ 2.4	46
9.80	12.92	30,000	3.26	2.61	1.608	3.34	+ 2.5	47
9.83	13.10	30,600	3.31	2.61	1.610	3.37	+ 1.8	48
9.84	13.06	30,400	3.28	2.60	1.611	3.35	+ 2.1	49
9.79	12.91	30,000	3.27	2.60	1.612	3.35	+ 2.4	50
9.81	12.88	29,900	3.25	2.59	1.612	3.34	+ 2.8	51
9.82	12.83	29,700	3.22	2.58	1.614	3.35	+ 4.0	52
9.83	12.88	29,900	3.24	2.58	1.615	3.35	+ 3.4	53
9.68	12.68	29,300	3.25	2.57	1.617	3.34	+ 2.8	54
9.72	12.79	29,600	3.26	2.57	1.617	3.34	+ 2.5	55
9.70	12.86	29,800	3.29	2.57	1.617	3.34	+ 1.5	56
9.65	12.87	29,800	3.31	2.57	1.617	3.34	+ 0.9	57
Total, Nos. 7 to 36 and 40 to 57, + 189.0; average, $\frac{189.0}{48}$							+ 3.9	

the formula,

$$C = \frac{Q}{b\,H^{1.5}} \qquad (9)$$

are given in Col. 11.

Comparison with Model Coefficients

Determination of Model Discharge Coefficients.—Discharge coefficients for the dam spillway were measured by the Bureau of Reclamation on a 1 : 72 scale model (39). It was found that because of the shape of the drumgate crest, which changed with the elevation of the gate, C, in Eq. 8 was neither a constant nor a function of the head. The Bureau found that the tests made on the model could be represented by the equation,

$$Q = K\,b\,H^{n} \qquad (10)$$

in which K and n are constants which vary only with the elevation of the gate crest. Values of K and n for each elevation of the drum-gate crest are given (39*a*) in Fig. 44.

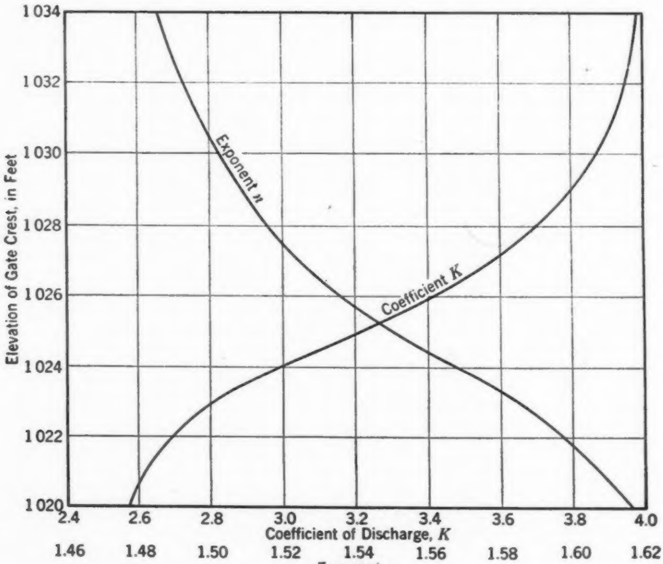


Fig. 44.—Values of K and n in Eq. 10

To compare the model tests with the results of observations on the prototype, it was desirable that coefficients of discharge be expressed in the same terms for each. The data obtained on the prototype do not cover a sufficient range to permit the determination of K and n. However, values of C can be computed from the Bureau's tests. Equating the two expressions for Q, Eqs. 8 and 10,

$$K\,b\,H^{n} = C\,b\,H^{1.5} \qquad (11)$$

From this,

$$C = K H^{n-1.5} \quad \text{.....................} \quad (12)$$

Values of K and n from Fig. 44 are given in Cols. 12 and 13 of Table 5, and the corresponding value of C, computed by Eq. 12, is given in Col. 14. This column has been headed C_m in order to distinguish the coefficients computed on the basis of the model tests from those determined on the prototype. Col. 15, Table 5, compares the two values of C by showing the percentage by which C_m is in error when compared to C.

A few of these values should not be used for drawing general conclusions. The first six observations are for relatively small discharges and low heads on the crest for which errors of observation cause large percentages of variation. Observations Nos. 37 to 39 give discharge coefficients for a single gate. They are probably correct but should not be compared with the coefficients determined for three gates, since the discharge over a single gate is subject to much greater end contractions. If these nine observations are eliminated, the average of the forty-eight remaining values is +3.9%, indicating that the model tests predicted discharges which average 3.9% greater than those actually obtained. The reason for this is not known, although it is possible that the model coefficients are not accurately known in this range. It is generally believed that, other things being equal, small-scale models will give discharge coefficients somewhat less than the prototype. There is no reason to discard any prototype observations other than those referred to, as all questionable observations were omitted before calculations were made. It is believed that the values of C determined for the prototype are reasonably accurate.

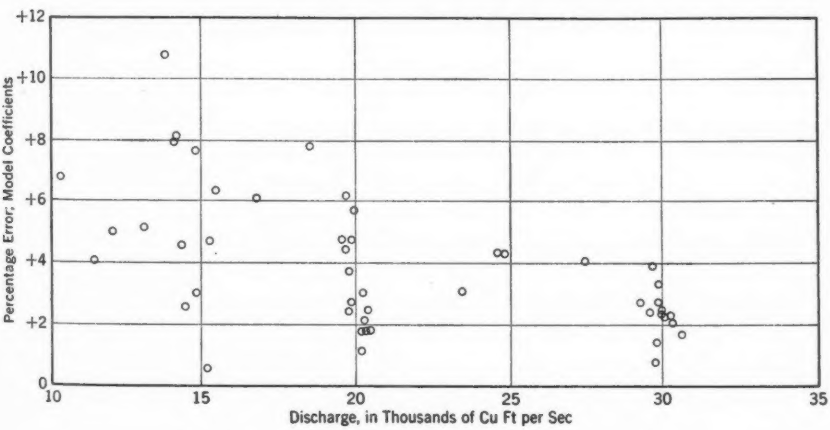


FIG. 45.—PERCENTAGE OF ERROR IN MODEL COEFFICIENTS

Fig. 45 shows the percentage difference of the forty-eight observations considered, plotted against the discharge. There is a definite tendency for the difference to decrease with increasing discharge. In this connection, it is interesting to note that, of the tests performed by the Bureau of Reclamation,

only five actually fell within the range of operating conditions represented by the observations on the prototype. In other words, the model discharge coefficients for this range have been determined largely by extrapolation. As the region of more model data is approached, the difference becomes less. It is possible that the difference may decrease still further for higher discharges. Observations for discharges in the range of 60,000 to 100,000 cu ft per sec will be necessary before a more definite comparison of the model and its prototype can be made.

Conclusion

The comparison shows that model tests may be relied on to predict prototype discharge coefficients with an accuracy at least equal to that of stream-flow measurements. The average discrepancy of 3.9% is not serious and is certainly less than might be expected without the benefit of model tests. It is quite possible that this difference might have been much less if the measurements on the prototype had been made in a range of discharges that were more adequately covered by model tests. This seems to be indicated by the trend of Fig. 45.

Accurate quantitative comparisons of model and prototype discharge coefficients are scarce because the model tests are generally made in the region of maximum discharge which seldom, if ever, occurs in the prototype. Many more comparisons could be made if the model were tested not only for the maximum discharge but also for the smaller discharges that occur more often. It is suggested that future tests on spillway coefficients be made to include the range of frequent prototype operation so that it will not be necessary to wait for the maximum flood to obtain verification of the model studies.

PRESSURE HEADS ON A WESTERN DAM

By J. C. Stevens,[9] M. Am. Soc. C. E., and R. B. Cochrane,[10] Assoc. M. Am. Soc. C. E.

Synopsis

The main hydraulic features pertaining to the design of a western dam were studied by models. Among these features were the hydraulic characteristics of the 50-ft by 50-ft crest gates with water flowing beneath them (40), and the hydraulic effect of departures of the profile of the constructed dam from the design profile. During the latter studies the pressures on the model crest were determined for various gate openings.

A series of piezometer taps, corresponding to those in the model, were placed. By means of these taps the pressure on the crest of the dam for various gate openings was observed. In addition, the surface profile of the prototype nappe was determined.

This paper presents the results of the pressure-head profiles on the underface of the overflowing nappe in both model and prototype. When the gate was in the downstream position, the pressure profiles of model and prototype checked fairly well; when the gate was in the upstream position, however, anomalies developed that may serve as the subject for some interesting speculations.

The Model

A replica of a 10-ft length of the crest and gate was made to a scale of 1 : 5 in a flume 2 ft wide, 10 ft deep, and 10 ft long (Fig. 46). In the prototype piers there were two gate grooves; in one the gate seal was 9 ft upstream from the center line of the crest and in the other it was 8 ft downstream. To simulate these positions the entire model crest block could be shifted to either position. The gate was held by stay rods so that it moved parallel to itself, and the rods were so long that this movement virtually simulated the prototype. The effect of the gate grooves was absent in the model, watertightness being secured by stanching strips of rubber.

Model Tests

The model tests were made primarily to determine the effect of slight departures of the constructed crest from the design crest, and to establish the tolerances that might be permitted in the crest and ogee profile. These experiments consisted essentially of observing pressures at the contact surface between

[9] (Deleted by censors.)

[10] (Deleted by censors.)

the concrete crest and the overflowing nappe. Three distorted crests were studied for gate openings of 0.25, 0.5, 1.0, 2.0, and 3.0 ft with the gate in both the upstream and the downstream positions. Fig. 47 shows the design crest and the three types of distortions used in the experiments.

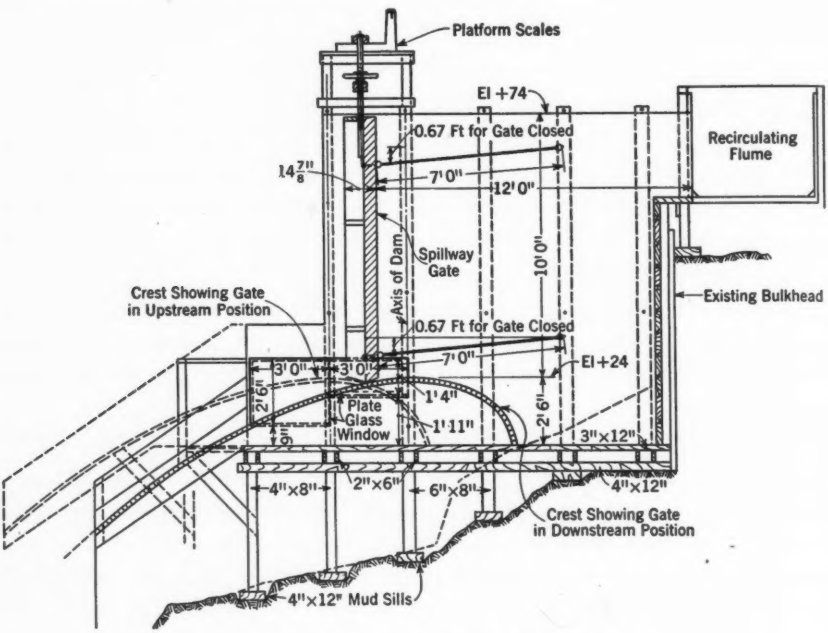


FIG. 46.—SECTIONAL ELEVATION OF GATE MODEL; SCALE, 1 : 5

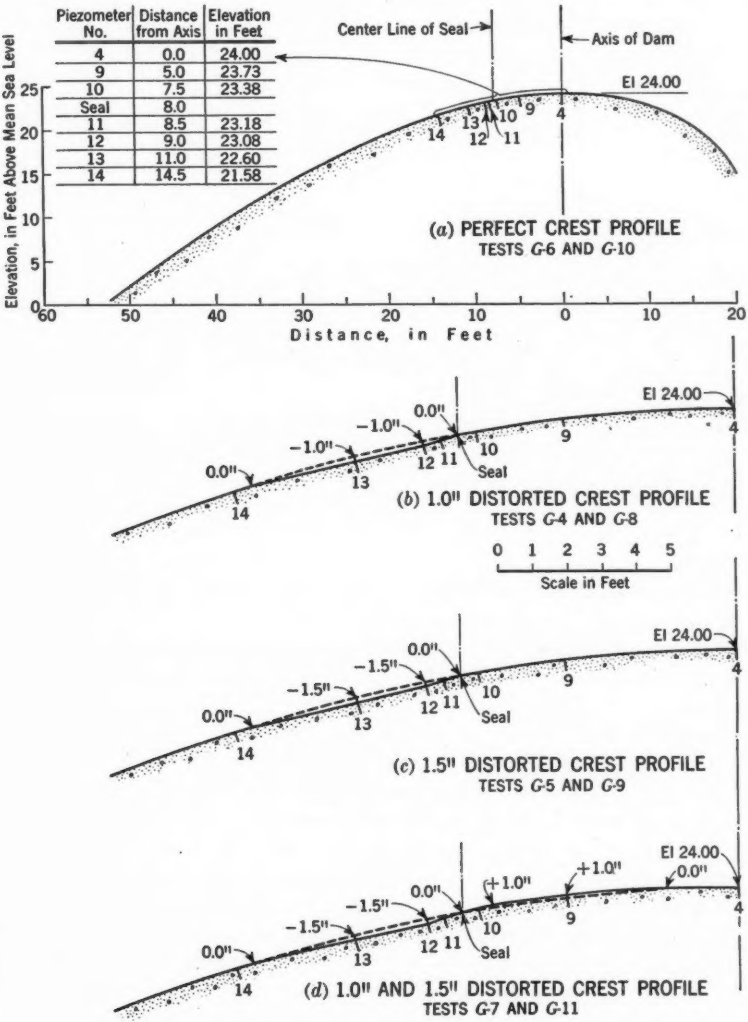
In addition to tests on the 1 : 5 model, pressure on the crest was observed for a large range of gate openings in the middle one of three bays of a 1 : 36 model. The gate was in the downstream position only, and the piezometer taps were at 5-ft intervals downstream from the center line of the crest. Since the prototype gate openings were 4 ft and less, comparisons up to that value only could be made.

PROTOTYPE TESTS

Prototype tests were made in Bay No. 13 of the completed dam in October, 1939. The crest profile in this bay was without appreciable departure from the design crest. The piezometer taps were spaced to conform to those of the 1 : 5 model except that in the prototype, Piezometer No. 1, 4 ft upstream from the axis, was substituted for model Piezometer No. 8, 9.58 ft upstream from the axis. The protoptype also had, in addition to those in the model, Piezometers Nos. 10 and 12 located 27.5 ft and 42.5 ft downstream from the axis.

The piezometer taps were made in a steel plate 10 in. wide set flush with the concrete and bent to the shape of the design crest. From each tap a $\frac{1}{4}$-in. iron pipe led through a stopcock to a manifold in an inspection gallery in the

dam. A 60-lb Bourdon pressure gage, reading positive pressures above atmospheric, was connected to the manifold at El. −28.89 ft, so that the pressure at each piezometer could be read in turn.



Piezometer No.	Distance from Axis	Elevation in Feet
4	0.0	24.00
9	5.0	23.73
10	7.5	23.38
Seal	8.0	
11	8.5	23.18
12	9.0	23.08
13	11.0	22.60
14	14.5	21.58

FIG. 47.—CREST PROFILES, MODEL OF DAM

Two series of tests were made, one with the gate in the downstream groove—its normal position—and the other in the upstream groove—an emergency position. In addition to observing the pressures at the underface of the nappe, the surface profile of the nappe was obtained for a distance of 24.5 ft down-

stream from the gate seal when in its downstream position and 13 ft downstream from the gate seal when in its upstream position.

TABLE 6.—WATER SURFACE ELEVATION (PROTOTYPE)

Distance from axis (ft)	Gate wide open	Opening, in Feet:					
		0.25	0.5	1.0	2.0	3.0	4.0
		(*a*) GATE 9 FT UPSTREAM					
−9.0	....	23.2	23.4	23.9	24.9	25.9	26.9
−3.3	59.2	....	....	....	....	....	....
−3.0	....	24.7	25.0	25.1	25.7	26.2	27.2
0	....	....	....	....	....	....	....
+3.8	56.0	....	....	....	....	27.0	27.8
+4.1	....	26.1	26.0	26.0	26.1	....	....
		(*b*) GATE 8 FT DOWNSTREAM					
+ 8.0	....	23.6	23.8	24.3	25.3	26.3	27.3
+12.8	50.8	....	....	....	....	....	....
+13.2	....	22.8	22.6	23.3	24.1	24.4	25.2
+23.1	....	19.6	19.5	20.2	20.6	21.5	21.2
+32.4	....	15.9	....	....	16.2	....	....
+32.5	....	....	....	15.6	....	....	....
+32.6	....	....	15.3	....	....	17.0	17.4

The surface profile for a length of 16 ft near the crest was also obtained for "gate removed," when the crest was under a head of 47.8 ft. There are no facilities for measuring the flow through any one bay, but tests on the 1 : 36 model indicate that under that head this 50-ft bay was discharging about 62,000 cu ft per sec or 1,240 cu ft sec per ft of crest.

The surface profiles (see Table 6) were obtained by means of a steel plate 48 in. by 48 in. by $1\frac{1}{2}$ in., weighing about 1,000 lb, handled by the auxiliary hoist on the gantry crane which could be moved perpendicular to the axis of the dam. There is much spray from the gate groove in the piers (see Fig. 48) beneath the gate which, however, did not affect the position of the steel plate; but when the corner of the plate touched the solid nappe the plate was kicked violently downstream.

During these tests the tailwater level was 12 ft so that the crest was not submerged.

COMPARATIVE RESULTS

The results of the tests on models and prototype are presented in Table 7, other data for a 2-ft gate opening being plotted in Fig. 49. Observations on the 1 : 36 scale model, shown in Fig. 49, were also made for gate openings of 1, 3, and 4 ft. For gate openings of 0.25 and 0.5 ft the comparison was between the prototype and 1 : 5 model only (see Table 7).

The pressure heads on both model and prototype are in fair agreement for the gate in the downstream position. The maximum negative pressure head observed in the prototype was −1.2 ft for a gate opening of 1 ft at Piezometer No. 6, just below the gate seal. The greatest negative heads in either model had about the same values at approximately the same localities.

For the gate in the upstream position there are some interesting departures of the prototype pressure heads from those of the model. Table 7(*b*) shows the comparative pressure heads (see also Fig. 49(*b*), for a 2-ft opening). Unfortunately, prototype Piezometers Nos. 3 and 8 were clogged. Every effort to free them with compressed air failed.

If the protype pressure heads for Piezometer No. 3 could have been taken for gate openings of 0.25 and 0.5 ft, one might assume that conformity could

have been demonstrated, but one dares not go any farther with that assumption. For gate openings of 1, 2, and 3 ft, the anomalies between prototype piezometer heads Nos. 2 and 3, and the corresponding model piezometer heads Nos. 4 and 9, became increasingly obvious.

In drawing the line of prototype pressure heads between Piezometers Nos. 7 and 9 for the five gate openings, one might have "upped" the curve to conform to the line obtained in the model, but without the model results the prototype line would not have been so drawn. Moreover, to have drawn it otherwise than indicated in the example (Fig. 49), particularly for the gate openings of 3 and 4 ft, would have made it a rather "goofy" curve.

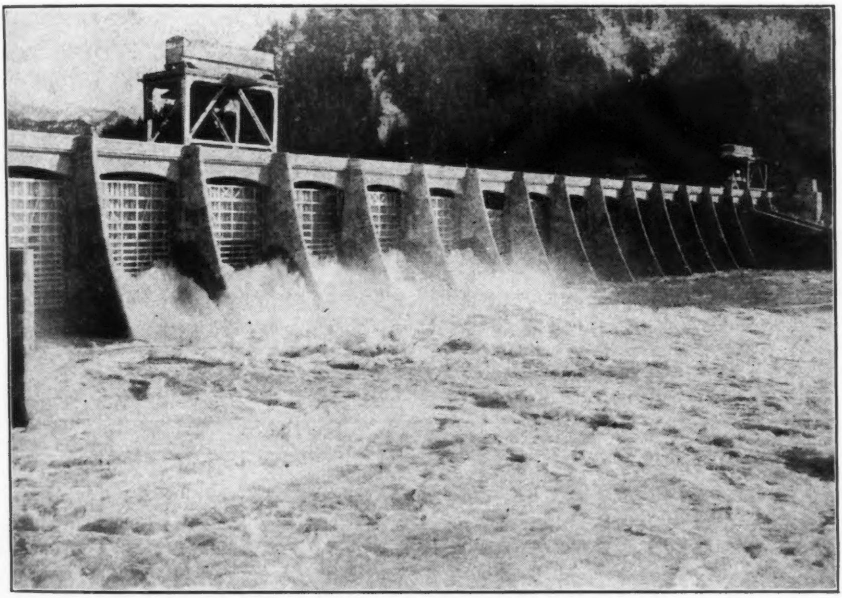


FIG. 48.—VIEW OF DAM SHOWING SPRAY FROM GATE GROOVE IN PIERS

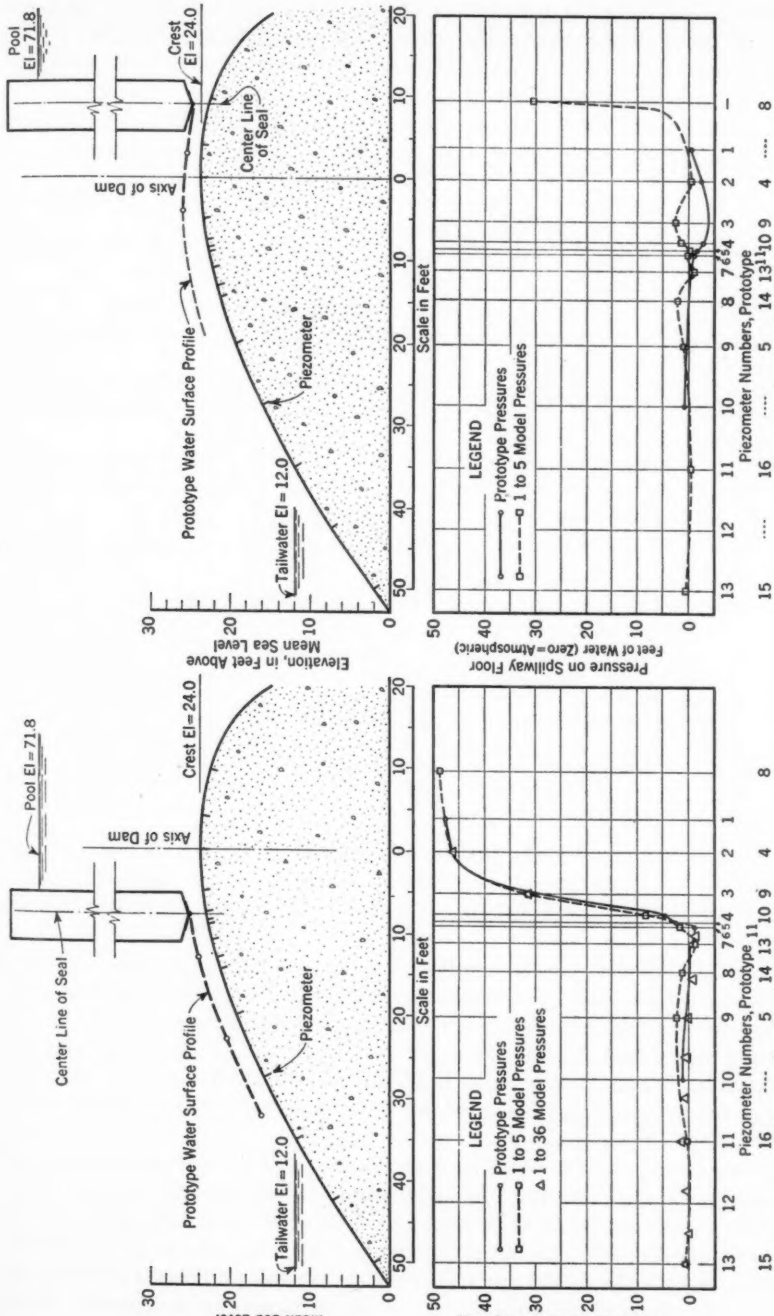
The explanation for the discordant results undoubtedly lies in the fact that with the gate in the upstream position there is a tendency for the jet to spring free from the crest. This tendency creates a zone of subatmospheric pressure just beyond the axis or crest of the spillway. The maximum observed negative pressure head in the prototype in this zone is −5.0 ft at Piezometer No. 4 (Table 7(*b*), item 26), whereas the projected pressure head at Piezometer No. 3 (Fig. 49) might have reached −6.0 ft.

These negative pressure heads appear to reach a maximum about 5 ft beyond the crest and gradually diminish to positive pressures about 15 ft beyond the crest (see Table 7(*b*), 4-ft opening). This effect doubtless would become more marked and reach a maximum at some larger gate opening, and thereafter gradually diminish, as evidenced in the last column of Table 7 where all pressure heads are definitely positive. For the prototype this is a

TABLE 7.—COMPARISON OF PRESSURE (IN FEET[a]) ON SPILLWAY FACE, MODEL AND PROTOTYPE

No.	Distance from axis (ft)	Structure	Piezometer No.[b]	Gate Opening, in Feet: 0.25	0.5	1.0	2.0	3.0	4.0	Wide open
(a) Center of Seal 8 Ft Downstream from Axis										
1	−9.6	Model	8	49.3	49.3	49.3	49.3	49.3	....	....
2	−4.0	Prototype	1	48.5	48.5	48.5	47.9	47.0	46.2	15.4
3	0	Model	4	48.0	48.0	47.6	46.6	46.8	....	....
4		Prototype	2	48.3	48.3	47.7	46.6	44.9	43.7	10.6
5	5.0	Model	9[c]	47.5	45.1	39.8	31.4	31.9	....	....
6	7.5	Model	10	22.8	14.9	9.6	8.1	11.7	....	....
7		Prototype	4	17.8	9.8	5.8	4.7	7.3	9.5	3.7
8	8.5	Model	11	0.9	1.3	1.8	2.7	5.4	....	....
9		Prototype	5	−0.1	0.5	1.4	2.5	4.9	6.0	5.2
10	9.0	Model	12	0.7	0.7	0.9	1.9	4.0	....	....
11		Prototype	6	0.3	0.3	−1.2	−0.9	0.6	0.6	5.0
12	11.0	Model	13	−0.8	−0.9	−1.0	−1.0	−0.4	....	....
13		Prototype	7	0.4	0.1	0.1	−0.5	−0.2	0.7	5.4
14	14.5	Model	14[c]	1.5	1.6	1.6	1.4	1.3	....	....
15	20.0	Model	5	1.3	1.6	2.1	2.5	2.6	....	....
16		Prototype	9	−0.6	−0.2	0.6	0.9	1.5	0.9	8.0
17	27.5	Prototype	10	0.1	0.4	1.3	1.3	1.3	1.6	9.0
18	35.0	Model	16[c]	0.4	0.4	0.4	0.5	0.5	....	....
19	50.0	Model	15[c]	1.0	1.2	1.2	1.1	1.1	....	....
(b) Center of Seal 9 Ft Upstream from Axis										
20	−9.6	Model	8	43.6	43.5	34.4	30.3	30.9	....	....
21	−4.0	Prototype	1	1.2	0.7	0.1	−0.5	−0.5	1.2	15.4
22	0	Model	4	0.2	0.4	0.2	−0.2	−0.2	....	....
23		Prototype	2	−0.1	−0.7	−1.3	−2.5	−3.5	−3.7	10.6
24	5.0	Model	9[c]	1.2	2.1	2.9	2.5	2.4	....	....
25	7.5	Model	10	0.4	0.9	1.5	1.4	1.2	....	....
26		Prototype	4	−0.3	0.0	−1.2	−3.0	−4.1	−5.0	3.7
27	8.5	Model	11	0.1	0.1	0.2	−0.1	−0.4	....	....
28		Prototype	5	0.2	0.2	−0.1	−1.6	−2.8	−2.8	5.2
29	9.0	Model	12	0.1	0.1	0.2	0.1	−0.1	....	....
30		Prototype	6	−0.3	−0.3	0.0	−1.2	−2.5	−2.7	5.0
31	11.0	Model	13	−0.1	−0.2	−0.4	−0.5	−0.8	....	....
32		Prototype	7	0.1	−0.2	0.1	−0.5	−1.7	−1.7	5.4
33	14.5	Model	14[c]	0.2	0.3	0.7	1.9	2.1	....	....
34	20.0	Model	5	0.2	0.3	0.4	0.9	0.8	....	....
35		Prototype	9	−0.2	−0.2	0.0	0.3	0.3	0.9	8.0
36	27.5	Prototype	10	−0.2	0.1	0.1	0.7	1.0	1.3	9.0
37	35.0	Model	16[c]	0.0	−0.1	−0.2	−0.4	−0.4	....	....
38	50.0	Model	15[c]	0.3	0.3	0.3	0.4	0.4	....	....

[a] Pressures are given in feet of water on the spillway floor (zero feet of water equals one atmosphere).
[b] See Fig. 49. [c] Corresponding prototype piezometer clogged.



(a) Gate in Downstream Position

(b) Gate in Upstream Position

FIG. 49.—SPILLWAY PRESSURE HEADS; 2-FT OPENING

perfectly normal behavior, and the line of pressure heads is believed to represent the true state of affairs. Why, though, did not the model experiments disclose the same tendency? All model pressure heads with the gate in the upstream position are on the positive side.

To show these anomalies better, Table 8 has been prepared. It gives the difference in pressure heads between prototype and model downstream from

TABLE 8.—DIFFERENCE BETWEEN PRESSURE HEADS FOR THE GATE IN THE UPSTREAM POSITION

(Prototype Value Minus Value Determined from Model)

Gate opening (ft)	PROTOTYPE (UPPER) AND 1 : 5 MODEL (LOWER) PIEZOMETER NOS.:											
	2 4	4 10	5 11	6 12	7 13	9 5	2 4	4 10	5 11	6 12	7 13	9 5
Distance[a]	0.0	7.5	8.5	9.0	11.0	20.0	0.0	7.5	8.5	9.0	11.0	20.0
	(a) MODEL TEST G-4						(c) MODEL TEST G-6					
0.25	−0.4	−0.7	0.1	−0.4	−0.3	−0.6	−0.3	−0.7	0.1	−0.4	0.2	−0.4
0.50	−1.1	−0.5	0.1	−0.4	−0.6	−1.1	−1.1	−0.9	0.1	−0.4	0.0	−0.5
1.00	−1.5	−2.1	−0.3	−0.4	−1.0	−0.7	−1.5	−2.7	−0.3	−0.2	0.5	−0.4
2.00	−2.2	−3.0	−1.2	−2.8	−2.3	−1.3	−2.3	−4.4	−1.5	−1.3	0.0	−0.6
3.00	−3.3	−3.5	−1.8	−4.1	−3.6	−0.7	−3.3	−5.3	−2.4	−2.4	−0.9	−0.5
	(b) MODEL TEST G-5						(d) MODEL TEST G-7					
0.25	−0.4	−0.7	0.1	−0.2	−0.1	−0.4	−0.5	−0.3	0.1	−0.3	−0.2	−0.4
0.50	−1.2	−0.6	0.1	−0.1	−0.4	−0.5	−1.2	0.0	0.1	−0.3	−0.6	−0.5
1.00	−1.6	−2.3	−0.3	0.2	−0.6	−0.4	−1.6	−1.3	−0.3	0.0	−1.0	−0.5
2.00	−2.3	−3.4	−1.3	−2.0	−2.2	−0.5	−2.4	−3.1	−1.9	−2.5	−2.7	−0.9
3.00	−3.3	−3.3	−2.0	−3.5	−3.8	−0.6	−3.3	−3.2	−2.8	−4.4	−4.6	−0.9

[a] Distance from axis of dam to piezometer, in feet.

the crest for the gate in the upstream position. Nearly all differences are negative; that is, the prototype pressure heads are lower than the model pressure heads, and these differences increase as the gate openings become larger.

Fig. 47 shows that model tests G-6 were made with an undistorted crest, while tests G-4, G-5, and G-7 were with distorted crests. The depressions in the concrete crest, however, begin at the downstream gate seal, which is 8 ft below the axis except for G-7 for which crest was raised between the gate seal and the axis. Anomalous pressure heads shown in Table 7(*b*) appear upstream of these crest distortions; hence, these distortions cannot be assigned as the cause of the discrepancies.

The negative pressures resulting from the nappe adhering to the underface of the gate below the seal (40) obviously cannot be assigned as a cause of the departure of the model from prototype behavior for two principal reasons: First, the anomalies occur far downstream from the gate and, second, the underface of both the model and the protype gate was perforated to admit air. The perforations on the prototype gate consist of three rows of staggered $1\frac{1}{16}$-in.

holes, so spaced as to be the equivalent to a continuous $\frac{7}{8}$-in. slot parallel to the rubber seal strip and communicating with the atmosphere on the downstream side of the gate. The first row of holes is 5 in. downstream from the center line of the seal strip. The perforations through the underface of the 1 : 5 model gate simulated those of the prototype as nearly as was practicable.

Conclusions

The writers can offer no valid explanation for the anomalous results between model and prototype with the gate in the upstream position.

Under such high velocities—45 ft per sec for the prototype and 20 ft per sec for the model—the difficulty of obtaining accurate pressure heads through piezometers was fully recognized and every precaution was taken to eliminate such sources of error. The departures from conformity are not of this nature. The model consistently refused to show negative pressures where such pressures should have obtained just below the crest.

METER MEASUREMENTS OF DAM DISCHARGE

By Edward Soucek,[11] Assoc. M. Am. Soc. C. E.

Synopsis

A comparison between a series of current-meter measurements of discharge over a dam on a midwestern river and tests of a 1 : 12 scale model of a section of the dam is presented in this paper. Prototype heads covered by the measurements ranged from 1.7 to 6.3 ft, and discharges ranged from less than 2,000 to nearly 16,000 cu ft per sec. Eleven of the twenty-three discharge measurements were made while the dam was submerged. Submergences ranged from less than 12% to nearly 90%.

Model discharges averaged approximately 5% less than comparable discharges obtained for the prototype. The comparison would appear somewhat more favorable if made in terms of head. It is by no means certain that the discrepancy is chargeable to the model.

Both model and prototype exhibit a considerably smaller diminution of discharge capacity, due to submergence, than was anticipated. The model tests demonstrate the inadequacy of single-valued relations between submergence and reduction in discharge capacity. Observations on the behavior of the nappe are presented.

Introduction

Submerged flow, defined as the condition that exists when the tailwater at a dam is at an elevation greater than that of the crest, often occurs during floods at low-head dams. Dams designed to remain submerged at all times are used to increase depths in lakes and rivers. Despite the considerable importance of the problem, published data on submerged flow over dams are exceedingly rare. The writer knows of no previous model-prototype comparison involving this phenomenon. These facts motivated the investigation described in this paper.

Prototype

Measurement of the dam at five cross sections revealed that it was built to close agreement with construction drawings. The river bottom upstream from the dam, although somewhat variable transversely, can be represented fairly well by the broken line in Fig. 50. The dam is founded on rock which, at the toe, has been excavated and scoured to a maximum depth approximately 20 ft lower than the crest.

The spillway is 273.50 ft long, composed of a main section 269.50 ft long and a section 4.00 ft long apparently intended for use as a fishway. These

[11] Senior Civ. Engr., U. S. Engr. Office, Wilmington, N. C.

sections are separated by a wall 1.48 ft wide that has its top 1.41 ft higher than the spillway crest. In profile, the top of the dividing wall roughly parallels the spillway. The wall is subject to overflow when the head on the dam exceeds 1.41 ft. An additional length of 6.75 ft is subject to overflow when the head exceeds 3.10 ft on the spillway proper. This additional length is a flat-crest section, 2.00 ft wide, at one end of the dam.

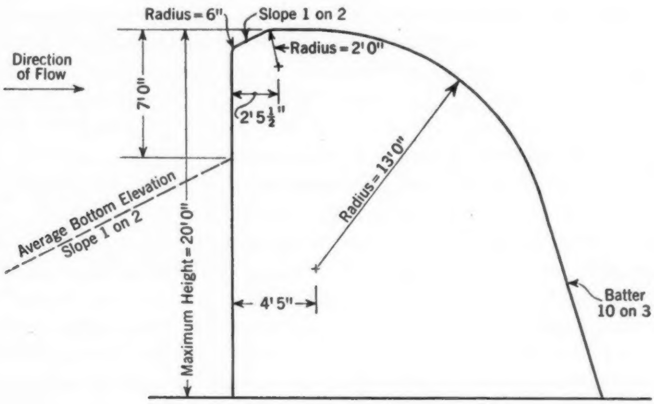


Fig. 50.—Spillway Cross Section

In comparing discharges of model and prototype, the measured prototype discharge was reduced by the estimated flow over the odd sections—namely, the dividing wall and the flat crest. Owing to the short length of overflow for the wall section and the low head on the high flat-crest sections, these corrections were small, never exceeding 1% of the measured discharge. Since any reasonable weir coefficient used in making the adjustment would give substantially the same results, the computation is not explained in detail, but both measured and corrected discharges are given.

The laboratory head-gate structures act as a non-overflow section of the dam 30 ft long. The gate pier adjacent to the spillway is well rounded, and no contraction of the nappe can be observed at the laboratory end of the dam. The power house forms a short non-overflow section of the other end of the dam. The intake crib, trashracks of which are parallel to the river bank, tends to reduce end contraction here also. Since even the Francis correction for a full contraction would have reduced the effective length of the spillway by only about 0.2% at the maximum head observed, no adjustment for end effects was considered necessary.

All data pertaining to the prototype are based on field measurements made during the course of the study.

Observations on Prototype

The head on the dam was read on a staff gage mounted on a retaining wall on the laboratory bank of the river. The gage is only 6 ft upstream from a line projected along the spillway crest, but its sheltered position, 30 ft trans-

versely from the overflow section, appears to keep the readings from being affected by drop-down. Some concern was felt over this apparently unfavorable location, but simultaneous observations of the water surface with a surveyor's level at the gage and at points clearly beyond any possible drop-down effect indicated that the location was satisfactory. The situation seems somewhat analogous to the measurement of head for a contracted weir in the side of a tank. There seems to be no reason why the head could not be measured nearly in the plane of the weir, as well as at some distance upstream, provided that the distance from the point of measurement to the edge of the weir were sufficiently great. If water approached the head-gate structures with a high velocity, it is conceivable that some "piling up" might occur which would increase the gage reading. Observations indicated the absence of any such phenomenon in the present case. Because of bank friction and the alinement of the river bank upstream from the gage area, the approach of water toward this vicinity is much slower than out in the center of the channel.

The tailwater elevation was measured by an automatic water stage recorder 350 ft downstream from the dam. There is a controlling riffle about 1,200 ft downstream which produces a flat slope in this reach at low stages. During the high discharges considered in this study, an ice jam which extended for several miles downstream from the dam exerted a similar effect. The ice jam built up to the dam on a number of occasions but was never within 500 ft of the dam during any of the measurements. These data on downstream conditions are cited because it is desired to show that the tailwater level would have been affected very little by changing the measuring point, provided that the point was clearly beyond the immediate effects of the dam, as was the case.

Discharge measurements were made from a cable station 470 ft downstream from the dam. Price current meters in good condition were used. Twenty of the measurements were taken by the 0.2 and 0.8 depth method. The remaining three were based on surface velocity alone. For these, a reduction coefficient of 0.9 was applied to determine the mean velocity in the vertical. (Some of the measurements made during the course of the study were used in making up the station record during the period of ice effect.) With due regard for the difficulty of making accurate measurements under adverse conditions, the measurements are considered reliable. A number of attempted measurements had to be abandoned because of floating ice.

Model Tests

The model was a 1 : 12 scale replica of the cross section of the prototype but did not reproduce the structure as regards length, end effects, or bottom irregularities. The model was made by stretching a galvanized iron skin plate tightly over a welded steel frame. The surface presented to the flowing water was very smooth.

The model was operated in a glass-sided steel flume 26 ft long, 2.5 ft deep, and 2.56 ft wide, glass to glass. The model was bolted to the floor of the flume 15 ft from its downstream end. End seals were of pitch and marine putty, smoothed to the profile of the model. Bottom seals were of portland cement mortar. The silt against the upstream side of the prototype was

simulated by an inclined plane of wood. Wooden baffles upstream from the model were used to produce uniform approach conditions.

Water was supplied to the flume at constant rates from the laboratory circulating system and was measured on a sharp-crested, fully suppressed and aerated, rectangular weir 2.51 ft long and 1.29 ft high. To insure an accurate rating, the weir was calibrated in place. This was considered especially necessary in view of the high discharges which taxed the capacity of the weir. A larger weir would have been preferable. For the calibration, a by-pass flume leading to a 20,000-lb capacity weighing tank was installed. An attempt was made to keep the condition of the weir crest the same during the model tests as during the calibration.

Prior to the tests, it was determined that the friction slope in the flume at the depths and discharges which were to be used would not exceed approximately 0.0001. Accordingly, the effect of friction in the flume could be neglected. This made it unnecessary to attain exact correspondence of points at which heads were measured on model and prototype, again provided that measurements were beyond the immediate effects of the dam. The head was measured 3.1 ft upstream from the back of the dam and the tailwater elevation 11.3 ft downstream from the same point. These points were determined experimentally as being out of the drop-down curve and beyond the rising and irregular water surface downstream from the dam, respectively. Both water levels were observed with point gages of the rack and pinion type mounted along the center line of the flume.

The discharge over the model was controlled with a gate valve in a 10-in. supply line leading to the weir tank. The tailwater elevation was regulated with a hinged flap gate at the downstream end of the flume.

Notation

The letter symbols used in this paper are defined where they first appear and are assembled for convenience of reference under "Bibliography and Notation" at the end of the Symposium.

Results of Model Tests

Fig. 51 shows the relation observed for unsubmerged or free discharge between the coefficient C and the energy head for the model in the formula:

$$Q = C\,L\left(H + \frac{V^2}{2\,g}\right)^{1.5} \quad \text{.................... (13)}$$

In this weir formula: Q is the time rate of discharge, in cubic feet per second; C is a coefficient, dimensionally the square root of an acceleration; L is the length of a weir or dam crest, in feet; H is the measured head on a dam, referred to the elevation of the crest, in feet; V = average velocity of flow; and g = acceleration due to gravity. The head corresponding to the average velocity of approach, in feet, is expressed as $\frac{V^2}{2\,g}$.

Eq. 13 is used to characterize the flow whether free or submerged. The ratio of the discharge which occurs when the flow is submerged to that which

would occur under the same energy head for free flow conditions is used as a measure of the diminution of the discharge capacity caused by submergence.

The results of model tests for submerged conditions are shown in Fig. 52. Following usual practice, the diminution in discharge capacity due to sub-

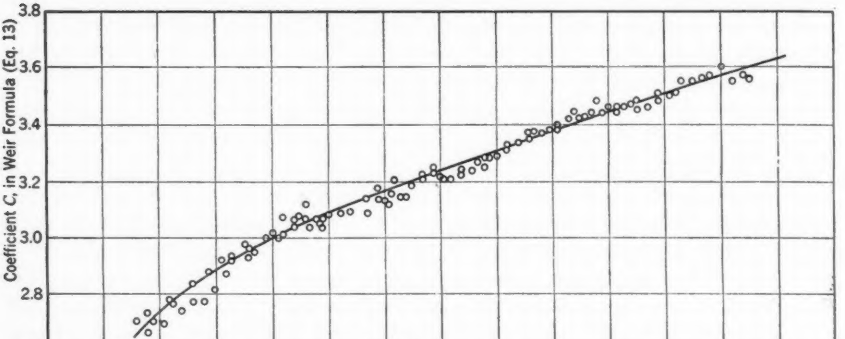


FIG. 51.—HEAD-COEFFICIENT RELATION FOR A MODEL WHEN NOT SUBMERGED

mergence is indicated by curves in which the submerged discharge, expressed as a percentage of the unsubmerged discharge which would obtain under the same energy head, is plotted against the submergence. For submergences lower than 45%, the relation between discharge and energy head was not affected by the submergence so the curves are not carried below a 45% submergence. The total model discharge and the discharge per foot of crest length are both given on these curves, the latter item being more significant since the length of the model bears only an incidental relation to the length of the prototype.

In Fig. 53, the curves of Fig. 52 are compared to a larger scale. An interesting relation exists between the curves for the five higher discharges. Given any two of the curves, or one curve and a well-defined point on another, the remaining three or four curves could be fixed by linear interpolation or extrapolation in terms of the discharge along the scale of relative discharges. Perhaps a slightly better fit to the plotted points in Fig. 52 could have been obtained if the curves had been drawn without regard for this property. It is seen that the curve for the lowest discharge does not correspond to the other five. The energy head for this discharge is less than 0.1 ft on the model. The matter is of great interest in connection with model-prototype relations at low heads because there is some question as to the behavior of the prototype for a corresponding discharge. It is regretted that this question could not be thoroughly investigated.

When water flows over a submerged spillway, the nappe may either plunge downward along the face of the spillway (as when the flow is unsubmerged) or it may flow away from the dam at or near the surface of the tailwater. The flowing nappe condition might be further subdivided according to the

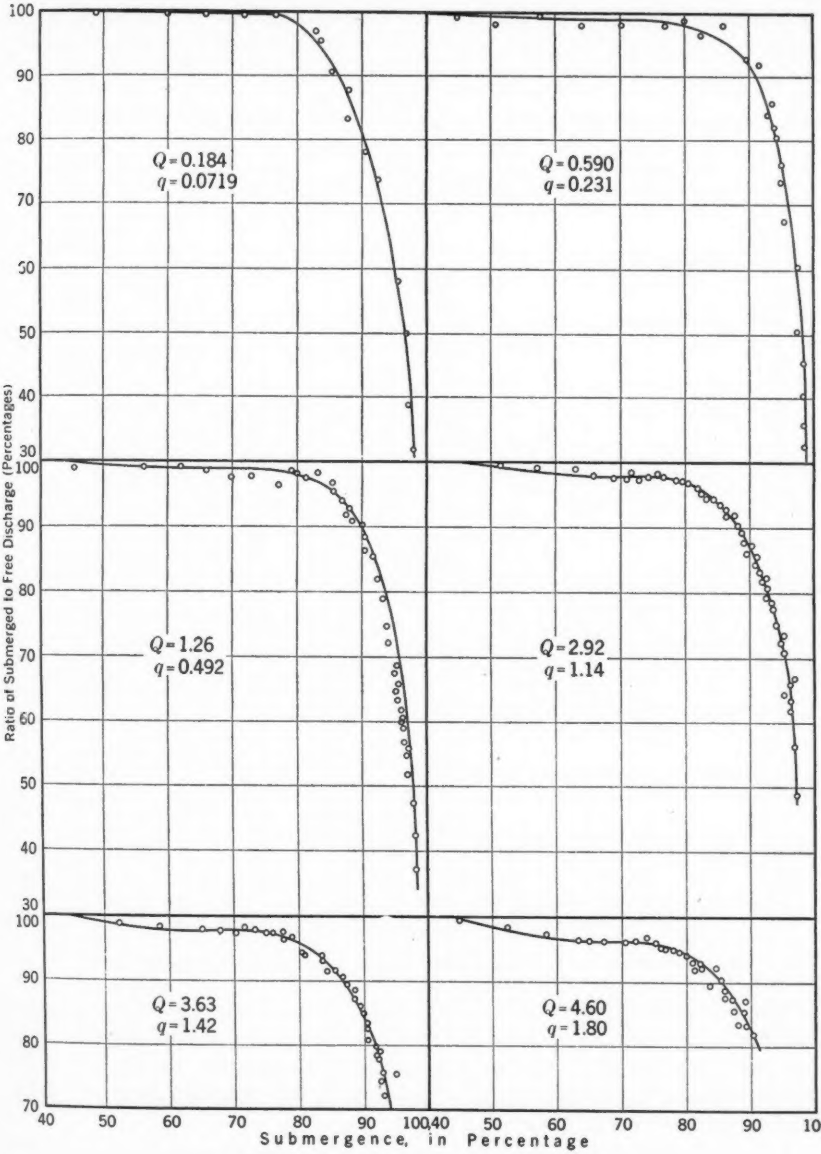


FIG. 52.—EFFECT OF SUBMERGENCE UPON MODEL DISCHARGE

appearance of the surface. Sometimes the surface has the appearance of a hydraulic jump. At other times there is a series of smooth standing waves. These conditions are of considerable interest, but in these tests the only distinction made was that between the plunging and flowing nappe conditions.

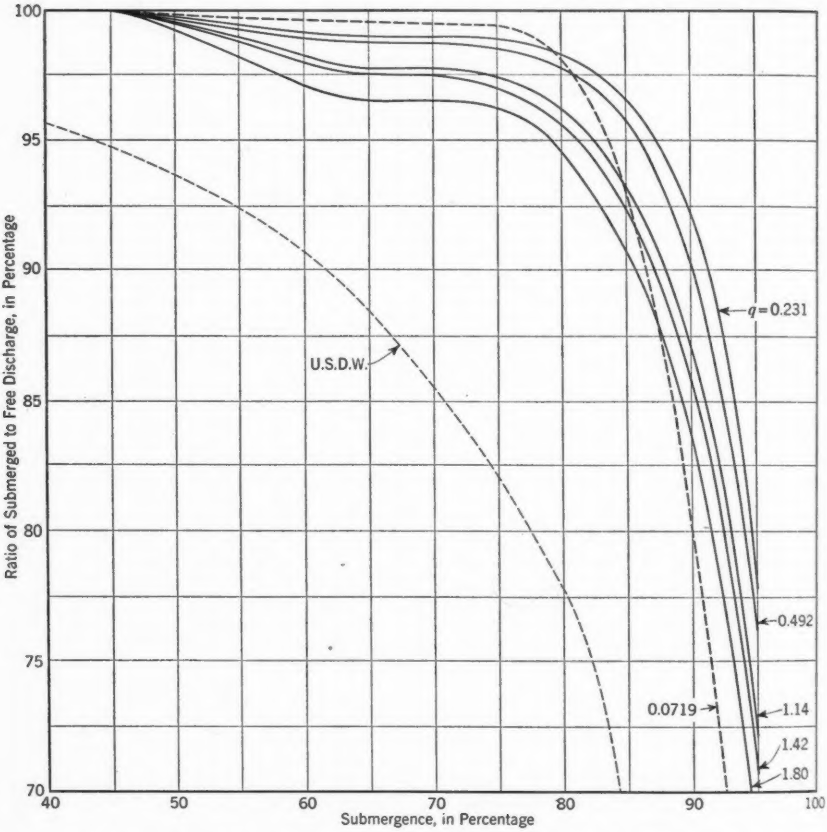


FIG. 53.—COMPARISON OF SUBMERGENCE CURVES FOR VARIOUS MODEL DISCHARGES

The plunging nappe is associated with low submergence, the flowing nappe with high submergence. For a constant discharge, the initial state tends to maintain itself so that there is a range of submergences within which the nappe may either plunge or flow, depending on the direction from which the indeterminate zone was entered.

Fig. 54 shows the results of tests made to determine the heads and tailwater elevations at which the changes from one nappe condition to the other occurred. The observational error was unavoidably high in making this determination. The results are self-explanatory. The lines drawn are not intended to represent the points but merely to give a better idea of the submergences at which these changes occurred.

Special tests were conducted to determine whether or not the behavior of the nappe had to be considered in studying the effect of submergence upon the discharge. This was done by manipulating the tailwater so as to create alternately plunging and flowing nappes for the same submergence and dis-

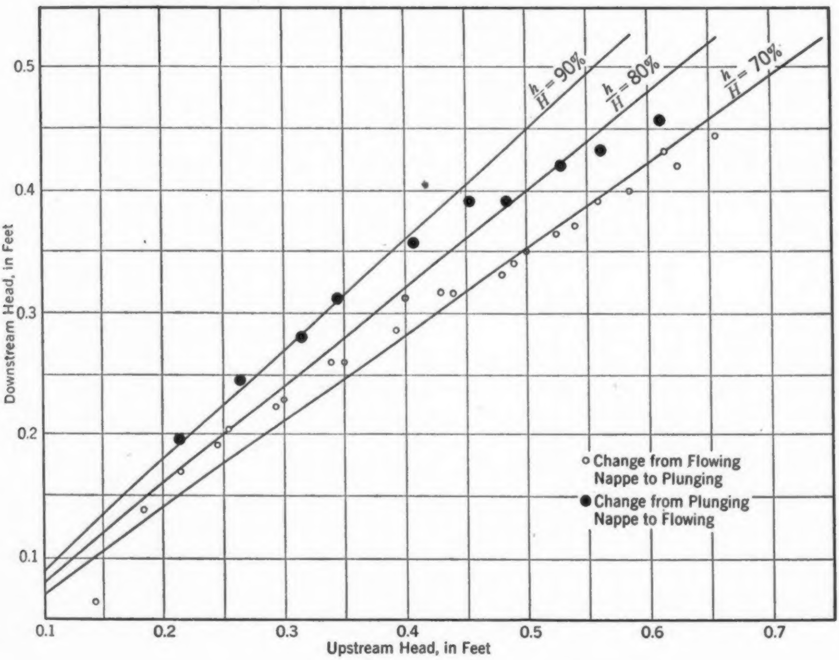


FIG. 54.—BEHAVIOR OF THE NAPPE OBSERVED ON THE MODEL

charge. The nappe condition had no apparent effect on the relation between submergence and discharge.

COMPARISON OF MODEL AND PROTOTYPE

Table 9 is a comparison of discharges measured for the prototype with those indicated by the model for corresponding conditions. The prototype measurements are arranged in ascending order of the discharge for free flow observations and in ascending order of the submergence for the submerged observations. The comparison could be made in various ways, the method adopted being one which gives the result in terms of discharge.

As previously explained, the length of the prototype spillway is 273.50 ft. A model 22.8 ft long $\left(\frac{273.50}{12}\right)$ would have been necessary to simulate the prototype as regards length. The model discharges per foot of length, therefore, must be multiplied by 22.8 as a length adjustment before being "scaled up" by Froude's law.

Table 9(*a*) gives prototype data which appear to require no explanation. Since model and prototype data are separated in the table, no subscripts are necessary to differentiate between them. Table 9(*b*) gives steps in the determination of the spillway discharge indicated by the model. Only the

TABLE 9.—COMPARISON OF MODEL AND PROTOTYPE

FLOW, Q (CU FT PER SEC)		HEAD (FT)				$\frac{H_t}{H}$ (%)	Energy head (ft)	(Energy head)$^{1.5}$	Coefficient, C	Discharge ratio (%)	Flow, q (cu ft per sec)	Flow, Q, indicated by model (cu ft per sec)	Difference (%)
Measured	Adjusted	H	$\frac{V^2}{2g}$	Energy head	H_t								
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
(*a*) PROTOTYPE DATA							(*b*) MODEL DATA					(*c*) COMPARATIVE DATA	
1,920	1,920	1.70	0.01	1.71	....	....	0.143	0.0541	2.86	....	0.155	1,760	− 8.3
1,960	1,960	1.74	0.01	1.75	....	....	0.146	0.0558	2.87	....	0.160	1,820	− 7.1
2,190	2,190	1.77	0.01	1.78	....	....	0.148	0.0569	2.88	....	0.164	1,860	−15.0
2,260	2,260	1.87	0.01	1.88	....	....	0.157	0.0622	2.90	....	0.180	2,050	− 9.2
2,410	2,410	1.96	0.01	1.97	....	....	0.164	0.0664	2.92	....	0.194	2,210	− 8.3
2,700	2,700	2.13	0.01	2.14	....	....	0.178	0.0751	2.95	....	0.222	2,530	− 6.3
2,910	2,910	2.18	0.02	2.20	....	....	0.183	0.0783	2.96	....	0.232	2,650	− 8.9
3,470	3,460	2.52	0.02	2.54	....	....	0.212	0.0976	3.02	....	0.295	3,360	− 2.9
3,840	3,830	2.64	0.03	2.67	....	....	0.223	0.105	3.04	....	0.320	3,640	− 5.0
4,500	4,490	2.96	0.04	3.00	....	....	0.250	0.125	3.08	....	0.385	4,380	− 2.4
6,180	6,160	3.57	0.06	3.63	....	....	0.303	0.167	3.17	....	0.529	6,020	− 2.3
6,580	6,560	3.69	0.07	3.76	....	....	0.313	0.175	3.18	....	0.557	6,340	− 3.4
3,450	3,440	2.39	0.02	2.41	0.28	11.7	0.201	0.090	3.00	1.000	0.270	3,070	−10.7
9,710	9,650	4.59	0.12	4.71	1.92	40.8	0.393	0.246	3.30	1.000	0.814	9,270	− 3.9
12,600	12,500	5.44	0.18	5.62	2.37	43.6	0.468	0.320	3.40	1.000	1.089	12,400	− 0.7
12,600[a]	12,500	5.19	0.19	5.38	2.45	47.2	0.448	0.300	3.37	0.998	1.008	11,500	− 7.9[a]
9,740	9,680	4.69	0.12	4.81	2.57	54.8	0.401	0.254	3.31	0.992	0.834	9,500	− 1.9
3,480	3,470	2.65	0.02	2.67	1.52	57.4	0.223	0.105	3.04	0.993	0.318	3,620	+ 4.3
15,900[a]	15,800	6.03	0.26	6.29	3.62	60.0	0.524	0.379	3.47	0.980	1.289	14,700	− 6.9[a]
14,750[a]	14,650	6.28	0.21	6.49	3.84	61.2	0.541	0.398	3.49	0.980	1.359	15,500	+ 5.9[a]
6,600	6,580	3.65	0.07	3.72	3.03	83.0	0.310	0.173	3.18	0.965	0.530	6,030	− 8.4
6,930	6,890	3.98	0.07	4.05	3.48	87.5	0.338	0.196	3.22	0.930	0.589	6,700	− 2.7
7,380	7,340	4.06	0.07	4.13	3.62	89.2	0.344	0.202	3.22	0.912	0.592	6,740	− 8.1

[a] Measurement by surface velocity with 0.9 reduction; other measurements, 0.2 and 0.8 depth.

energy head and the submergence in this part of the table are based on prototype observations. A brief explanation of the computation procedure seems desirable. The energy head for the model is obtained by dividing the corresponding prototype item by 12. The submergence, being a ratio, is numerically equal for model and prototype.

For free flow, or when the submergence is less than 45% (below which value the model indicated that submergence had no effect), q, the discharge per foot length of the model, is computed from the formula,

$$q = \frac{Q}{L} = C \left(H + \frac{V^2}{2g} \right)^{1.5} \qquad (14)$$

the coefficient being taken from the curve of Fig. 51.

When the submergence exceeds 45%, the discharge computed by Eq. 14 must be reduced by the curves of Fig. 53. The model shows, however, that

the ratio of submerged to free discharge at the same energy head (hereinafter called simply the "discharge ratio") is a function not only of the submergence but also of the discharge, which is as yet unknown. Accordingly, the final value of the discharge ratio cannot be found until the discharge is known and vice versa. A solution can be found by a few easy approximations.

A simple procedure is to assume that the discharge ratio has the value of 100%. A first trial value of q can now be found, and a second trial value of the discharge ratio can be read from Fig. 53 by interpolation. This gives a second trial value of q. The process is repeated until q remains unchanged by further approximation, which it will after a very few trials. Obviously, the procedure can be shortened if a close estimate of either q or the discharge ratio is made initially. No trial computations are shown in Table 9, but the correctness of the results can be verified without approximation. An easier method of calculation could be derived if a large number of discharges were to be determined.

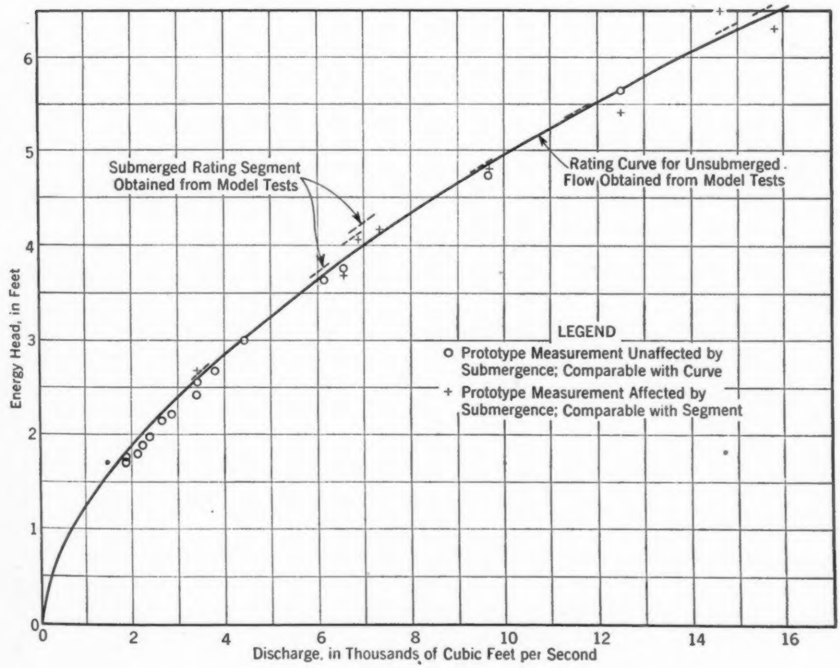


FIG. 55.—COMPARISON OF MODEL AND PROTOTYPE DISCHARGE

Cols. 13 and 14, Table 9, are comparative data. The spillway discharge indicated by the model is the product of three factors—the discharge per foot of length of the model, the length adjustment as previously explained, and the five-halves power of the scale ratio. Col. 14, Table 9, gives the percentage difference between the measured net spillway discharges and the

spillway discharges indicated by the model for corresponding energy heads and submergences. The difference is considered negative when the indicated discharge is less than the measured discharge.

In Fig. 55 the performance of the model is compared graphically with prototype measurements. The continuous curve is an energy-head-discharge relation for free flow over the spillway, established from the model tests. Observations on the prototype when unsubmerged, or when the submergence did not exceed 45%, are comparable directly with this curve.

To avoid confusing the diagram, continuous curves were not drawn for submerged flow. For each submergence at which a measurement was made, a short segment of the submerged rating indicated by the model tests was drawn at the corresponding energy head. Since submergence reduces the discharge for a given energy head, the submerged rating segments all lie above and to the left of the curve for free flow. It should be noticed that the three measurements which show the greatest discrepancies are based only on surface velocity. No close agreement can be expected for these three measurements.

Comment

Effect of Submergence on the Discharge.—Undoubtedly the best known tests of submerged flow over a rounded-crest dam are those performed for the U. S. Board of Engineers on Deep Waterways at Cornell University in Ithaca, N. Y. (41)(42), in 1899. Results of these tests are frequently quoted. Drawings of the dam on which the tests were made are quite generally available, so no illustration is included in this paper.

A part of the relation between submergence and the ratio of submerged to free discharge obtained on another dam is shown for comparison in Fig. 53. It is seen that the effect of submergence obtained in the present study was only a small fraction of that obtained on the dam in question. The effect of submergence on the model of a second dam was not appreciable until the submergence exceeded 45%, whereas the first dam was affected by much lower submergences.

The comparison is presented merely with the idea of showing that the use of "typical" submerged flow data can be nothing but a rough approximation. The use of the data on one dam to estimate the effect of submergence on the other would have caused greater errors than would have occurred by entirely neglecting the effect of submergence.

Analysis of the Effect of Submergence.—The data quoted in the foregoing section plot as a smooth curve. This fact appears to have given rise to a prevalent belief that, in general, the effect of submergence on the discharge over a dam can be represented by a single-valued curve relating submergence and the discharge ratio. Attempts to plot submerged spillway data in this manner invariably lead to a wide scatter of observation points. This, in turn, leads to the opinion that submerged flow over dams is a far more unstable phenomenon than is really the case. The submerged flow tests of the first dam were made at a constant head of 6.6 ft. This made the tailwater elevation and the submergence functions of the discharge, which had to be decreased as the tailwater was raised to keep the measured head constant. Study of

Fig. 53 shows that, if a set of observations were plotted, in which the submergence was functionally related to the discharge, a single curve rather than a family of curves would express the results adequately.

If a stable control exists downstream from a dam, the submergence will indeed be related to the discharge in this special way. If model tests are made for such a case, the tailwater must be regulated to represent the prototype condition if a single-valued relation between submergence and its effect on the discharge is desired. This procedure is not recommended. If unwilling to vary the tailwater independently of the discharge as a matter of curiosity, the experimenter should consider the possibility that the control might change and invalidate the results if these results were so restricted in scope. The ice effect which caused the unusual degree of submergence observed in this study is a pertinent example.

Measurement of Tailwater Elevation.—The Society's *Manual of Engineering Practice No. 11,* entitled "Letter Symbols and Glossary for Hydraulics," defines submergence as follows:

> "The ratio of the tail-water elevation to the head-water elevation, when both are higher than the crest, the overflow crest of the structure being the datum of reference. The distances up stream or down stream from the crest at which head-water and tail-water elevations are measured are important, but have not been standardized."

In connection with measurement of headwater elevation, there seems to be no reason for departing from the procedure commonly used for unsubmerged weirs and dams—namely, the measurement at a point sufficiently distant from the overflow to be safely out of the drop-down curve.

G. N. Cox, M. Am. Soc. C. E. (45), measured tailwater elevation very near the dam. At this point, especially when the nappe plunges, there is a decided depression in the water surface. Even for the flowing nappe condition, the water level near the dam is at a lower elevation than farther downstream. The reason for a negative slope in this reach is a readjustment of velocity distribution which has an effect that is similar, as regards energy transfer, to the recovery of pressure head which occurs in the diffusing cone of a venturi meter. The investigation by Professor Cox dealt primarily with the use of the submerged weir as a measuring device. For this purpose, the use of a tailwater measuring point very near the dam is apparently satisfactory.

For most purposes, however, it is believed that a tailwater measuring point clearly beyond the rising water surface is decidedly preferable. Professor Cox found it necessary to consider the behavior of the nappe and gave a different formula for each condition. This is a complication that should be avoided if possible. If recording gages are used, the behavior of the nappe cannot be determined definitely from gage readings alone if the submergence is in the critical zone. At times during observations on the second dam, a plunging nappe existed over part of the spillway length and a flowing nappe over the remainder.

The model tests indicated that if the tailwater level is measured at a sufficient distance from the dam, the behavior of the nappe need not be con-

sidered. This is just another way of stating that the over-all loss of head in flow over the model did not depend to any measurable extent on the behavior of the nappe.

It seems best, also, to consider the rising water surface downstream from the dam as a part of the phenomenon of submerged flow. Otherwise, the magnitude of the rise in water level must be estimated in fitting results of model tests to a stage-discharge relation downstream from the dam site. No satisfactory method of making this adjustment is available. It is probable that model tests indicate its magnitude more accurately than it can be computed.

Comparison of Model and Prototype.—On the whole, the agreement between model and prototype is considered satisfactory. Not much weight can be given to the three measurements made by surface velocity. If the coefficient 0.85 had been used for these measurements instead of 0.90, two of them would agree very well with the model. A coefficient of 0.85 is well within the range of possibility, but the originally estimated value of 0.90 has been retained. As may be seen from Fig. 55, the agreement between model and prototype except for the three surface velocity measurements is about as good as that between the discharge measurements themselves.

For estimating flood discharges from water level observations at dams, it would appear that the use of small-scale sectional models must be regarded as being as satisfactory as the current meter—from the standpoint of efficiency, far more so. In no case would an estimate of the backwater due to the dam, based on the model tests, have been in error by more than a few inches.

For all but two of the twenty-three measurements, the model indicated a discharge smaller than that measured on the prototype. To what extent this difference is chargeable to disproportionately large viscosity, surface tension, friction, and end effects is not known. Overregistration of the current meters may have accounted for some of the difference.

As previously suggested, the nonconformity of the curve for the lowest discharge in Fig. 53 to the general pattern for higher discharges is of special interest. It seems possible that the model fails for a discharge as low as this, but it cannot be disproved or substantiated from data now available.

Conclusions

The conclusions presented are necessarily tentative. However, they are considered to constitute a logical interpretation of the results of model tests and measurements of the dam.

(*a*) The effect of submergence on the discharge is affected by the shape of the spillway and perhaps by other factors. Data at present available are insufficient for generalization of the effect of submergence in terms of spillway shape. The use of "typical" data cannot be recommended.

(*b*) In general, it is not possible to express the effect of submergence upon the discharge by a single-valued relation between the submergence and the discharge ratio. In the range covered by these tests, the effect of submergence is greater for high discharges.

(c) If the tailwater elevation is measured at a sufficient distance from the dam, it is not necessary to consider the behavior of the nappe in determining the effect of submergence upon the discharge. For this and other reasons, the tailwater elevation should be measured beyond the immediate effects of the dam.

(d) Reasonably good agreement may be expected between spillway models and prototypes of the sizes used in these tests under submerged as well as free flow conditions.

Certain other observations might be presented as secondary or incidental conclusions, but these appear to have been covered to the extent that their importance justifies in the body of the paper.

Acknowledgment

Acknowledgment must be made of the assistance of many engineers. The possibilities of this study were recognized by E. W. Lane, M. Am. Soc. C. E., when an ice jam below the dam created unprecedented tailwater conditions. The prototype measurements were conducted by the writer under his direction. Some discharge measurements were made by the staff of the U. S. Geological Survey, under the direction of R. G. Kasel, M. Am. Soc. C. E., in connection with its regular stream-gaging work. The experiments on the model were conducted by C. L. Morgan, Jun. Am. Soc. C. E., under the joint direction of F. T. Mavis, M. Am. Soc. C. E., and the writer. The manuscript has been reviewed by F. W. Edwards, M. Am. Soc. C. E., and C. J. Posey, Assoc. M. Am. Soc. C. E., both of whom made valuable suggestions. Others who assisted in the project are: E. R. Van Driest, J. S. McNown, C. K. Willey, H. D. Brice, Juniors, Am. Soc. C. E., and G. L. Whitaker and B. D. Lind.

DEVELOPMENT OF MISSISSIPPI RIVER CHANNEL

By Frederick R. Brown,[12] Jun. Am. Soc. C. E.

Synopsis

The U. S. Waterways Experiment Station recently received from the U. S. Engineer Office, St. Louis (Mo.) District, field data that served to confirm the results obtained from a hydraulic model study (53) (54) conducted at the Experiment Station during 1936 and 1937 on a reach of the Mississippi River. A careful analysis of these data has been made and a comparison of the action of the prototype with the indications of the model furnishes the basis for this paper.

The Problem

The model study involved the development of an effective plan for improving the navigation channel of the Mississippi River. At a certain point the Mississippi River enters a rock gorge, the bluffs of which confine the flow of the river to a preceding section. At the latter point the rock bluffs on the left bank of the Mississippi River recede, allowing the river to widen and to divide its flow. The natural tendency of the river is to follow the chute channel. However, because of the more satisfactory navigation conditions that would prevail if the river could be made to develop the crossing to the right bluff line, two pile dikes were constructed in 1939 from the left bank. These dikes effected considerable improvement in the navigation channel, but did not eliminate the need for periodic maintenance dredging through this crossing. The material dredged from the crossing was spoiled in front of, and off the end of, one dike and in the chute behind this dike in an attempt to confine the river flow to the crossing. Samples obtained after the completion of the model study indicated that the river-bed material in the vicinity was composed of sand, gravel, and coarse rock fragments.

The Model

The scale ratios, model to prototype, were as follows: Horizontal dimensions, 1 : 600; and vertical dimensions, 1 : 150. The model was of the movable bed type, with banks and overbank topography molded in concrete and the movable channel section molded in crushed coal. In addition to the slope scale of 4 : 1 induced by the difference in scale ratios, the model was given a slight additional slope sufficient to provide adequate movement of the bed material used. The scales used were within the range in which it has been found that velocity distributions and current directions can be reproduced accurately in models of wide, shallow rivers such as the Mississippi.

[12] (Deleted by censors.)

After a thorough adjustment and verification of the model, in which operating conditions were developed such that the movement of bed material in the model was consistent with that in the prototype, tests of plans were begun The testing procedure consisted of installing the elements of a proposed improvement plan in the model, the bed of which had been molded to conform to the 1936 prototype conditions, and operating the model in accordance with the stages of a modified form of the 1935 hydrograph (Fig. 56). The 1935

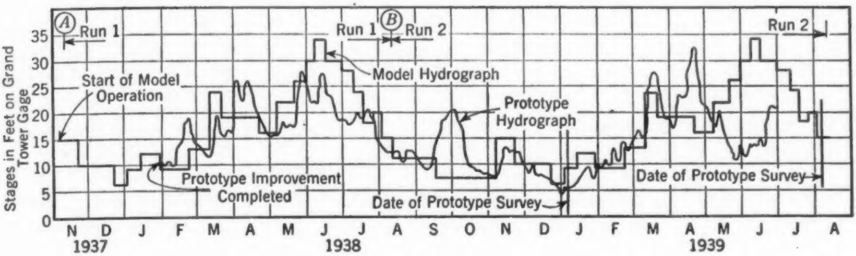


FIG. 56.—COMPARATIVE HYDROGRAPHS, MODEL AND PROTOTYPE

hydrograph included within its range an adequate distribution of critical high stages as well as normal and low stages. Therefore, since it was believed to embrace all conditions of flow likely to occur in the reach in question in the future, this hydrograph was selected for use in the model study. Both the dredging proposed in the improvement plan and the necessary maintenance dredging were simulated in the model.

It was thought advisable to begin testing the improvement plan at a stage comparable to that expected in the prototype at the time of completion of the prototype improvement. It was believed that this completion would occur in November, 1937. Consequently, operation of the model was started with the 15-ft stage in November (modified 1935 hydrograph). Although dredging of the initial pilot channel in the prototype was not actually completed until January 30, 1938, the effect of the difference in time was not considered to be important.

MODEL AND PROTOTYPE IMPROVEMENT PLANS

The plan included the shortening of one dike and the addition of a 1,500-ft trail dike downstream from this dike, a dredge cut 250 ft wide to a depth of − 13 ft (low-water reference plane) along the right bank of an island, use of the dredge spoil chiefly to build up the left bank of the dredge cut, and riprap and mattress protection for the left bank of the dredge cut and for the trail dike.

In the prototype, the outer 600 ft of one dike was removed, the 1,500-ft trail dike was constructed, and revetments were placed along the left bank of the new channel during the period from August, 1937, to January, 1938. The pilot cut for the re-alined channel was made during several dredging periods—namely, September 28, 1937, to January 30, 1938; March 17 to May 28, 1938; and August 20 to October 31, 1938. This cut was more than 400 ft wide in the prototype in contrast to the 250-ft wide pilot cut in the model. It is under-

stood that the dredge spoil (about 3,400,000 cu yd) was disposed generally in accordance with plans followed in the model test. In addition to the works scheduled in the improvement plan, prior to the August, 1939, prototype survey, the dike on the right bank was extended about 500 ft across the old channel. This extension was intended to aid the filling of the old channel.

Comparison of Prototype and Model Results

In general, the prototype reproduced accurately the results indicated by the model study. It should be stated that some divergence of action is to be expected, because, to the time of the August, 1939, prototype survey, the duration of the high stages (above 20 ft) in the prototype was less than half the duration of like stages in the model (Fig. 56). Available data indicate no greatly divergent tendencies at present between model and prototype. The district engineer reports that as of November, 1939, the maximum available depth through the old channel was only 5 ft, whereas project depth existed through the new channel, showing that the prototype is conforming sensibly to the indications of the model. Careful examination of prototype and model data reveals the following interesting comparisons:

(*a*) Until the time of maintenance dredging in January, 1939, the re-alined channel in the prototype exhibited a persistent tendency to shoal. This agrees with the model results, since by the end of Run 1 the model channel had shoaled to a controlling depth of 5 ft below the low-water reference plane.

(*b*) Since the prototype maintenance dredging of January, 1939, the scour in the re-alined channel indicated in the model has occurred also in the prototype, although at a somewhat slower rate. This difference in rate of scour may be explained by the fact that the plan developed in the model was based on the assumption that the river bed in this vicinity was composed of average river sand containing only a small amount of larger material, whereas actual spoil samples (obtained when the actual improvement was undertaken) were found to contain 40% rock fragments and large gravel.

(*c*) A tendency to scour was noted in the model at the upper end of the old channel along the right bank, and tests indicated that this channel would continue to fill after each high stage. Thus far, alternate scouring and filling has also occurred in the prototype, with scouring predominating. It is to be noted that development of the re-alined prototype channel is not yet complete. It is believed probable, however, that, once this new channel has fully developed, the old channel will become filled substantially as predicted in the model.

The comparison of model and prototype behavior in the reach of the Mississippi River involved will be supplemented from time to time as additional prototype data are made available.

HYDRAULIC CONDITIONS AT APPROACH TO A LOCK AND DAM

By William J. Hopkins,[13] Assoc. M. Am. Soc. C. E.

Synopsis

The new lock and dam here considered is located on the Monongahela River. Because of local conditions, the structure, as originally designed, was subjected to adverse currents that had an effect on the maneuverability of river vessels in the upper approach to the locks. At moderately high stages, the main currents, after sweeping around two comparatively sharp bends upstream, followed the lock side of the river, increased in velocity as they reached the upper approach, and then swung sharply across the lock entrance toward the center of the stream before discharging over the spillway section. Examination of the site and of the current conditions disclosed that the flow characteristics might have been due to several factors, or to a combination of any of these—namely, (*a*) the effect of the upstream bends on velocity and current distribution; (*b*) the effect of the hydrography in the vicinity of the approach on velocities and currents; and (*c*) the effect of a contraction in the stream cross section due to the lock and to the existence of a large bar along the shore opposite the lock chamber.

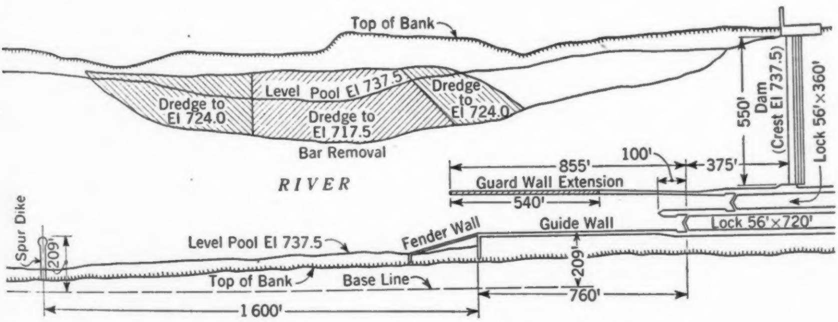


Fig. 57.—Modified Design of Upper Lock Approach

The existence of the foregoing condition prompted an investigation to be made by means of hydraulic model tests to study the causes of the current phenomena at the upper approach and to determine the most suitable means of improving the entrance conditions. On the basis of these studies, necessary

[13] (Deleted by censors.)

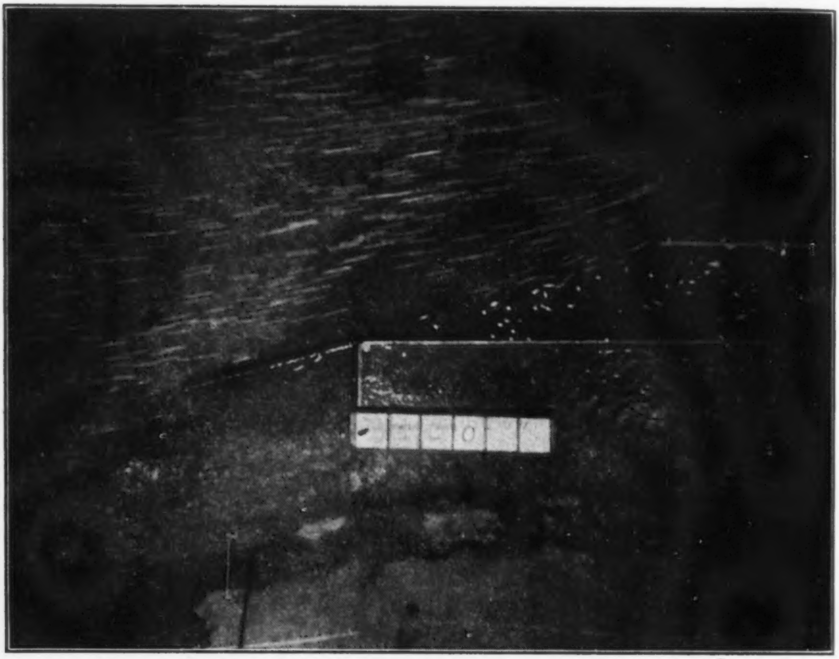


FIG. 58.—CONFETTI MOVEMENTS SHOWING CURRENT DIRECTIONS AND VELOCITIES; ORIGINAL DESIGN

FIG. 59.—SHIFT OF MODEL TOW DUE TO CURRENTS IN THE ORIGINAL DESIGN

revisions were made in the upper approach to the lock and in the stream channel above the dam. These involved an extension to the guard wall, the construction of a spur dike upstream from the lock, and the removal of a portion of the bar opposite the lock chamber.

To determine the accuracy and the fidelity with which the model simulated the prototype conditions, certain field tests were conducted. The results of these investigations showed that, generally, hydraulic similarity between the model and prototype existed. As far as can be ascertained from the experience of navigators in traversing the upper approach to the lock chamber, the flow conditions have been considerably improved since the modifications in the upper pool were effected.

Description of the Structure

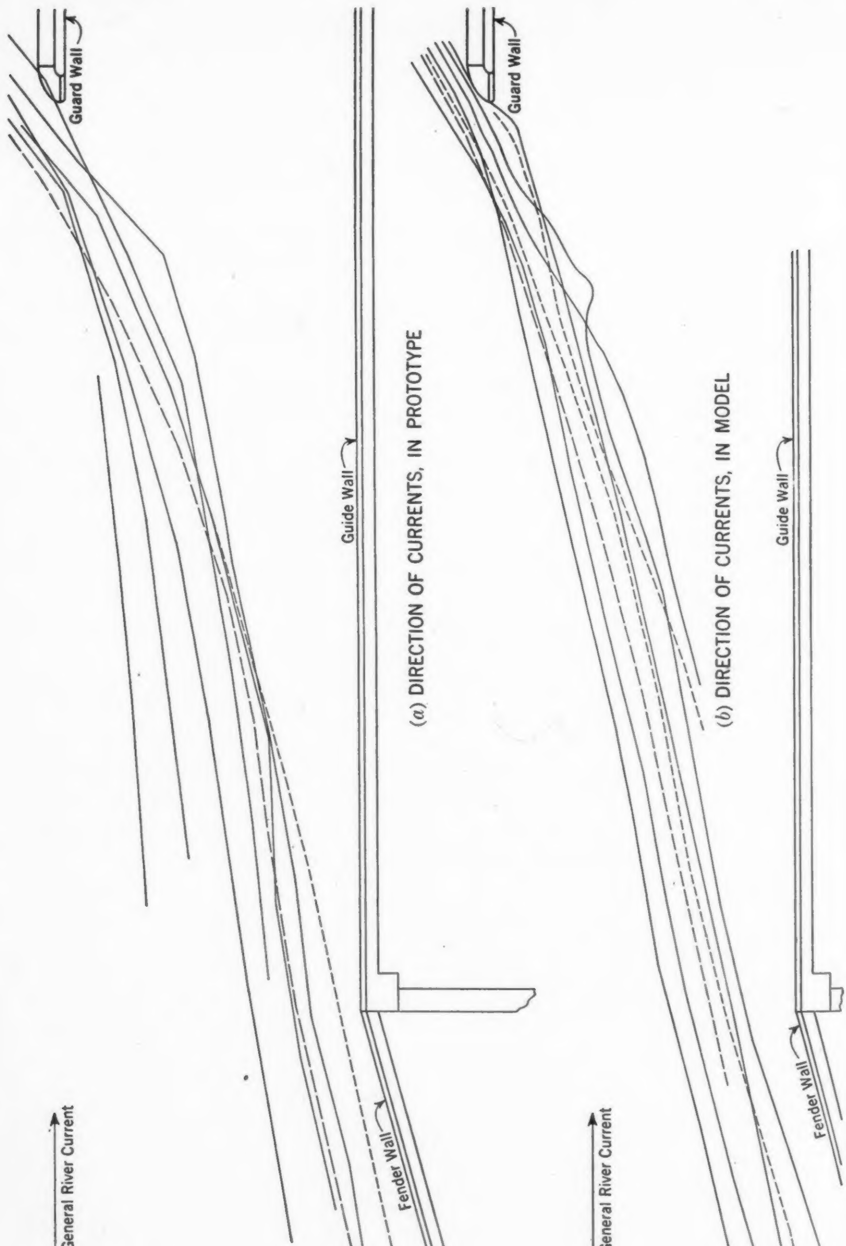
The dam is about 550 ft long from the left abutment to the lock, and is of the concrete gravity type. A modified, uncontrolled, ogee spillway section, with crest at El. 737.5 (mean sea level), is provided.

To accommodate the heavy river traffic in this vicinity, two locks having a lift of 10.6 ft are provided along the right bank of the stream. The land lock is 56 ft by 720 ft, and the river lock 56 ft by 360 ft. The tops of the guide and guard walls are at El. 749.0. As altered to produce suitable entrance conditions to the locks from above, the middle lock-wall and the guide and guard walls extend approximately 100 ft, 760 ft, and 855 ft, respectively, upstream from the upper gates of the land lock. In addition, a spur dike is provided at a point 1,600 ft upstream from the end of the guide wall. The dike is normal to the flow of the river and extends riverward from the shore to a point in line with the river face of the guide wall. It consists of a shell of steel sheet piling, backfilled for strength and stability. The general details of the structure, and the location of the component parts, are shown in Fig. 57.

The Model Tests

Approximately 2.4 miles of the river were reproduced in the model, the major part of the reach being upstream from the structure to develop accurately the flow characteristics at the upper approach. The model was undistorted, on a scale of 1 : 200, and was of the fixed-bed type built according to standard laboratory practice. All necessary details that might affect flow phenomena were reproduced faithfully to scale, including the lock and spillway and the topography in the immediate vicinity. The assumed channel roughness was reproduced as accurately as was physically practicable.

All basic tests for the design of the required improvements were made with the upper pool 0.5 ft below the top of the lock-walls. The flow corresponding to this condition was assumed to represent the greatest discharge under which the navigation facilities could be used. The tests consisted of analyses of the resulting velocities and currents, and of their effects on tows entering the locks from above. The investigations involved a determination of the flow condi-



(a) DIRECTION OF CURRENTS, IN PROTOTYPE

(b) DIRECTION OF CURRENTS, IN MODEL

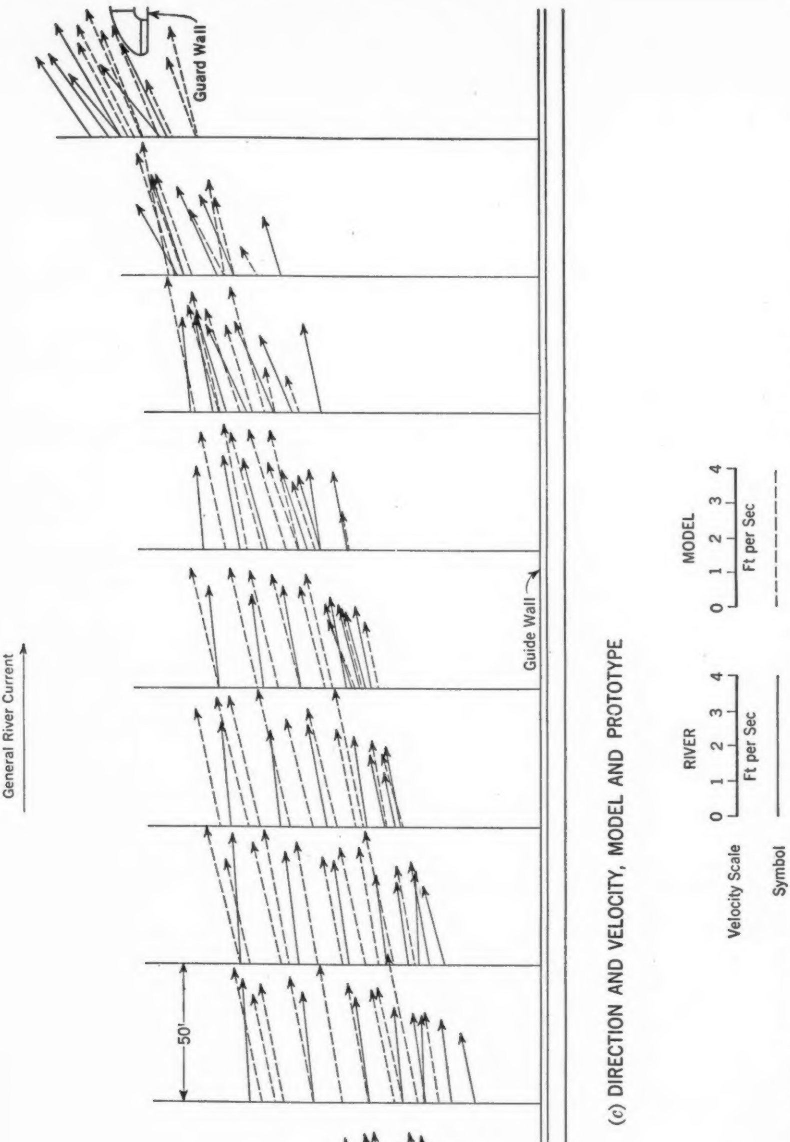


FIG. 60.—COMPARISON OF OBSERVATIONS ON SURFACE CURRENTS IN THE MODEL AND THE PROTOTYPE, FOR THE ORIGINAL CONDITION OF THE RIVER AND UPPER LOCK APPROACH

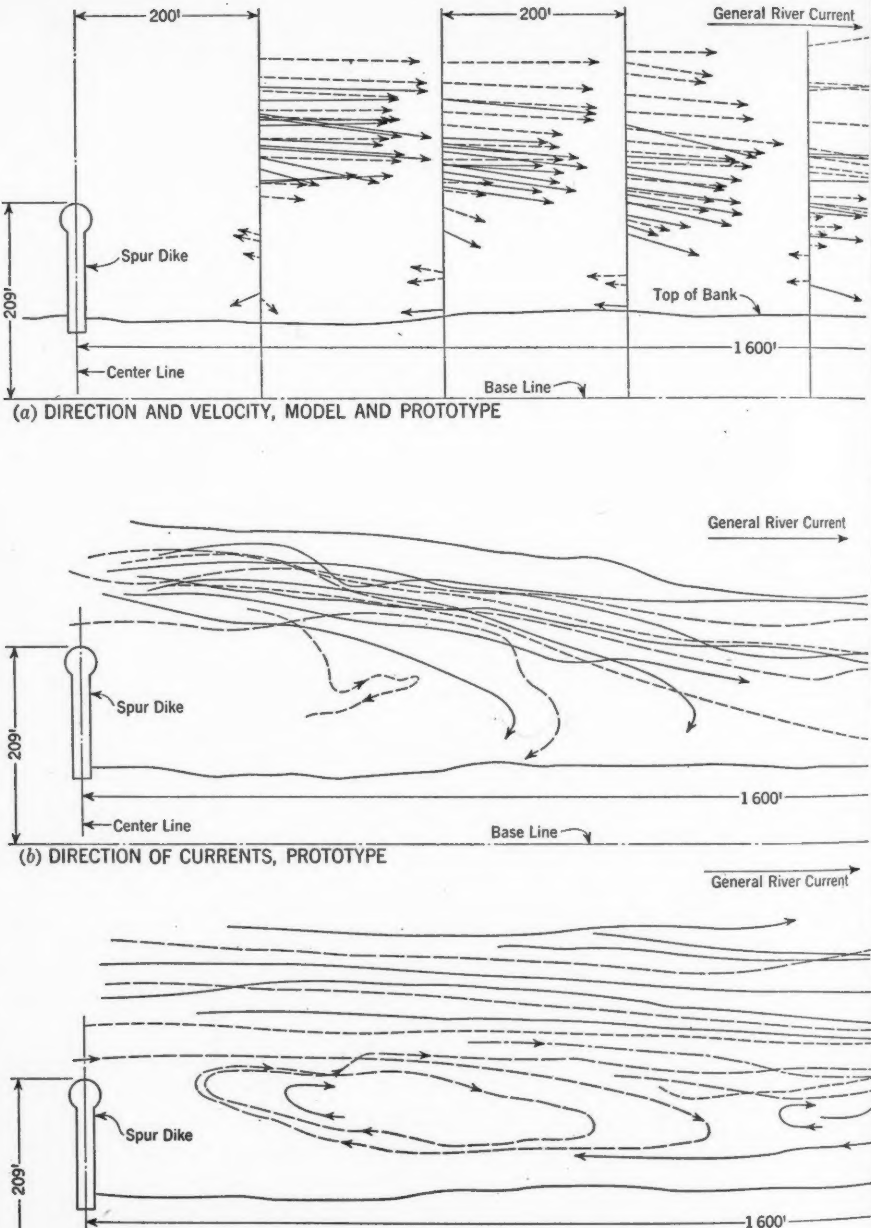
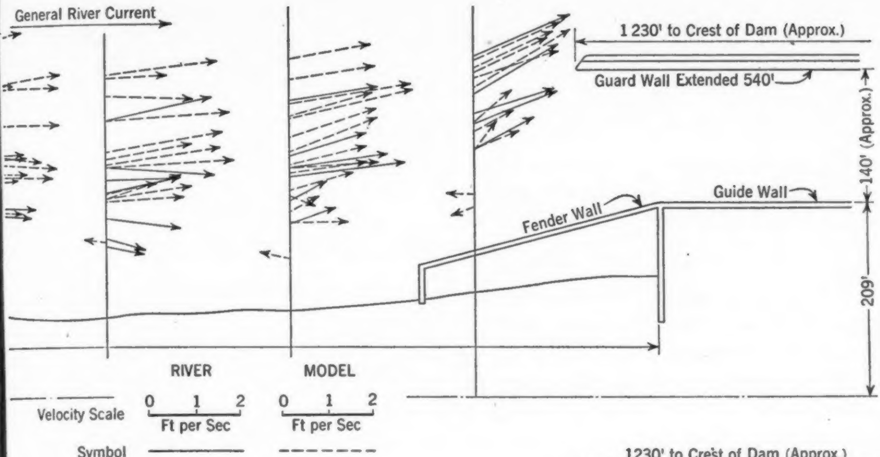
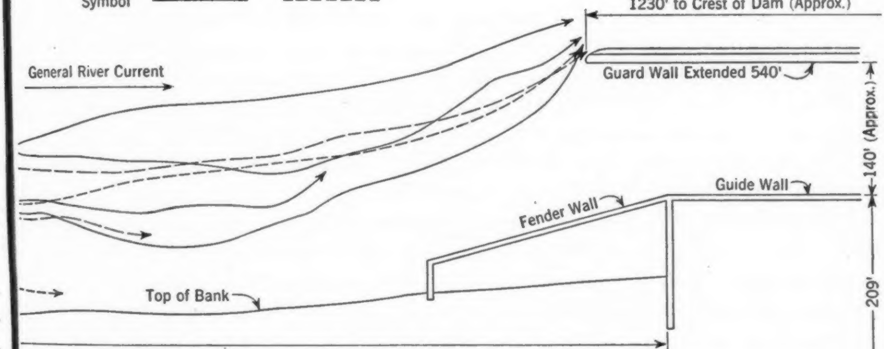
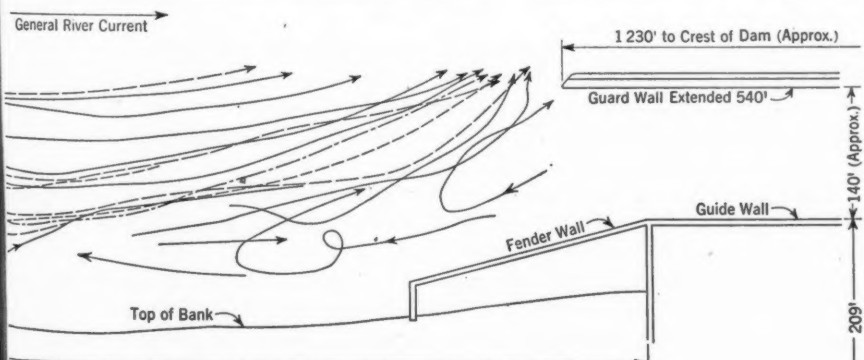


FIG. 61.—COMPARISON OF OBSERVATIONS ON SURFACE CURRENTS IN THE MODEL AND





SURFACE CURRENTS – UPPER APPROACH – PROTOTYPE



SURFACE CURRENTS – UPPER APPROACH – MODEL

Prototype for the Altered Conditions of the River and Upper Lock Approach

tions for the original design, and of the separate and combined effects due to extension of the guide and guard walls, dredging in the upper lock approach, dredging of the bar opposite the lock chamber, and deflection of the flow by means of dikes of various proportions and located in various positions.

The data obtained from the model were of three types:

(*a*) Notes on visual inspection of the flow conditions;

(*b*) Photographic records of confetti movements and the paths traversed by model tows; and

(*c*) Actual measurements of the surface velocities in the critical areas.

In the analysis of the test data for the several proposed designs, the photographs disclosed the alinement, direction, and velocity of the currents by the alinement, direction, and length of the streaks. In addition, the photographic record revealed the effect of these currents on the travel of model tows that were allowed to glide toward the lock without artificial maneuvering. For a true comparison of the data in the various designs, the latter procedure was standardized by the use of a tow-projecting device that imparted to each tow the same initial velocity and direction. Samples of the photographic data for the original design are shown in Figs. 58 and 59. The surface velocity measurements were made in the region where they were believed to have the greatest bearing on the maneuverability of tows. This was accomplished by spanning the critical region with a piece of plate glass and marking thereon the location of floating paper disks at regular intervals of time.

To provide data for a comparison with the prototype results, and thereby furnish a means of verifying the model, special tests were run on the model for both the original and altered conditions of the prototype structure. These data were in the form of photographs of the current movements and an analysis of the surface velocities in the upper approach, the procedure during these tests being the same as the foregoing.

The Field Tests

The details of the structure, both in the model and the prototype, were the same, so that an exact comparison could be made for verification, except for the effects of surface tension and channel roughness—factors which were impossible of exact simulation in the model. Field tests were made both before and after the alterations had been made to the prototype.

The field data were necessarily confined to measurements of current directions and surface velocities. No data on tow movements, as affected by the flow in the approach, were obtained, since the artificial guiding of the tows in the prototype could in no way be simulated by tow movements in the model. The current directions and velocities were obtained by tracing the paths of floats in the river and recording their locations at regular intervals of time. This was accomplished by spotting the floats simultaneously with three surveyor's transits located at strategic points. Corrections were applied to the measurements for wind velocity by noting the wind effect on floats in the lock chamber.

Verification of Results and Conclusions

A comparison of the data obtained on the model and in the field, in the form of current alinement and velocity vectors, is presented in Figs. 60 and 61. For the original design of the approach (Fig. 60), the operating conditions in the model and prototype were identical, corresponding to a head of 5.6 ft on the crest of the dam and to a discharge of 30,000 cu ft per sec (headwater elevation, 743.1). Operation for the revised design of the upper approach was not identical in the model and prototype. The head on the crest of the dam for this condition was 6.6 ft in the prototype as against 6.0 ft in the model, corresponding to flows of 40,000 and and 35,000 cu ft per sec, respectively (headwater elevations were 744.1 and 743.5 ft, respectively). The model tests had been conducted before an opportunity had presented itself for obtaining field measurements approximating the conditions tested in the model for the altered design, so that identical operation was not possible. However, the differences in head and discharge are not believed to be of sufficient magnitude to invalidate the results of the verification.

The degree of similarity between the model and prototype results is apparent from an examination of the data presented. Exact simulation by the model of the prototype phenomena was not attained; nor was it expected. Generally, the model data reveal close approximations for the magnitude of the prototype velocities, for the alinement of the currents, and for the nature and location of the quiescent pool formed downstream from the spur dike. A comparison of the data also shows a considerable reduction in the velocities of flow across the lock entrance and at the head of the guard wall, this condition existing in both the model and prototype.

In the course of the model and field tests, it was noted that slight surges in the flow occurred, especially in regions where an attempt was made to alter the flow characteristics, or where an obstruction to flow existed. Because of these surges, the resulting path of flow through any definite point for successive intervals of time was not identical, although the general pattern of flow for any area was approximately the same. A slight variation in the data should be expected, therefore, both in the model and prototype, and a comparison of the data should be made on the basis of general values and flow patterns rather than on the specific analysis of a particular flow line. Generally, therefore, the velocities, currents, and eddies in the model and prototype were similar.

Conclusion

It can be concluded that satisfactory verification of the model tests for the entrance conditions to the upper approach of the new lock and dam resulted. Subsequent experience in the maneuvering of tows in the reach of the stream in question has indicated it to be devoid of any serious hazards and to be satisfactory in every respect, which further attests to the success of the plan of alteration proposed as a result of the model tests. As in many other instances, model analysis in this case provided a direct approach to a problem that could not be solved by purely analytical methods.

BIBLIOGRAPHY AND NOTATION

By the Symposium Authors

I. Bibliography

Those who are closely associated with model-prototype work realize the few references that are available on the subject. The following list, including references in the text of the various papers, is probably far from complete,[13a] but is offered as a representative beginning:

(1) **"First Annual Report of Special Committee on Hydraulic Research,"** *Civil Engineering*, March, 1935, p. 169.

(2) **"Superelevation and Bridge Pier Losses in Model and Field,"** by J. G. Jobes and J. H. Douma, *ibid.*, January, 1943, p. 16.

(3) **"Hydraulic Design of Drop Structures for Gully Control,"** by B. T. Morris and D. C. Johnson, *Proceedings*, Am. Soc. C. E., January, 1942, p. 17.

(4) **"The Similarity of Motion of Water Through Sluices and Through Scale Models: Experiments with Models of Sluices of the Assuan Dam,"** by H. E. Hurst and D. A. F. Watt, *Proceedings*, Inst. C. E., London, Vol. 218, 1923–1924, pp. 72–112.

(5) **"Uberfallversuche in verschiedener Modellgrösse,"** by F. Eisner, *Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. 11, 1931, p. 416.

(6) **"Über Schwingungen von Wehren,"** by R. Seifert, *Zeitschrift des Vereines Deutscher Ingenieure*, Vol. 84, No. 7, February 17, 1940, pp. 105–110.

(7) **"Distinctive Features of Grand Coulee Outlets,"** by Jacob E. Warnock, *Civil Engineering*, December, 1940, pp. 779–782.

(8) **"Models Predict Behavior of Falling Dam,"** by A. J. Ackerman, *ibid.*, July, 1932, p. 415.

(9) Discussion by T. T. Knappen of **"Construction and Testing of Hydraulic Models, Muskingum Water-Shed Project,"** by G. E. Barnes and J. G. Jobes, *Transactions*, Am. Soc. C. E., Vol. 103 (1938), pp. 249 *et seq.*

(10) **"Comparison of Sluice-Gate Discharge in Model and Prototype,"** by F. W. Blaisdell, *ibid.*, Vol. 102 (1937), p. 544.

(11) **"Report on Experiments with a Model of Khesano Regulator on the Rohri Canal, Mile No. 194,"** by W. E. Bushby and Hiranandanio, *Technical Paper No. 1*, 1937, Government of Sind, Public Works Dept., India.

(12) **"Rapport sur les Travaux Effectués Pendant les Années 1932–35,"** by C. Camichel, *Travaux Scientifiques*, August, 1935.

(13) **Study of Hydrotechnical Questions by Means of Laboratory Researches on Reduced Scale Models. Comparison of the Results of Such Researches with Those of Direct Observation of the Natural Phenomena, with a View of Ascertaining How Far the Law of Similitude Is True,"** by C. Camichel and L. Escande, *Paper No. 57*, 3d Communication, 15th International Cong. of Navigation, Venice, 1931.

[13a] New references contributed by discussers will be added for the *Transactions* printing.

(14) **"La Similitude des ouvrages courts; experiences sur la grande forme de radoub du port du Havre,"** by C. Camichel, F. Bear, and L. Escande, Académie des Sciences, Seance du Lundi 29, August, 1932.

(15) **"Verification of Drop-Inlet Hydraulic-Model Studies by Field Tests,"** by E. R. Dodge, *Civil Engineering*, August, 1941, p. 496.

(16) **"Observational Hydraulics in Miami Conservancy District,"** by C. H. Eiffert, *ibid.*, November, 1939, p. 672.

(17) **"Über die Ähnlichkeit bei flussbaulichen Modellversuchen,"** by H. A. Einstein and R. Müller, *Schweizer Archiv für angewandte Wissenschaft und Technik*, Vol. 8.

(18) **"L'Étude sur modéle réduit des ouvrages de rupture de charge,"** by L. Escande, *Genie Civil*, December 16, 1939.

(19) **"Performance of Norris Stilling Basin During 1937 Flood,"** by G. H. Hickox, T. V. A., Knoxville, Tenn., July, 1938.

(20) **"Study of Hydrotechnical Questions by Means of Laboratory Researches on Reduced Scale Models. Comparison of the Results of Such Researches with Those of Direct Observation of the Natural Phenomena, with a View to Ascertaining How Far the Law of Similitude Is True,"** by Giulio de Marchi, *Paper No. 59*, 3d Communication, 15th International Cong. of Navigation, Venice, 1931.

(21) **"Experiments on Discharge Over Spillways and Models, Keokuk Dam,"** by F. A. Nagler and A. Davis, *Transactions*, Am. Soc. C. E., Vol. 94 (1930), p. 777.

(22) **"Observed Effects of Geometric Distortion in Hydraulic Models,"** by K. D. Nichols, *ibid.*, Vol. 104 (1939), p. 1488.

(23) **"Checks on the Model Law for Hydraulic Structures,"** by M. P. O'Brien, *Transactions*, Am. Geophysical Union, National Research Council, April 28–29, 1932, pp. 390–394.

(24) **"Conformity Between Model and Prototype Tests—Madden Dam Spillway,"** by P. S. O'Shaughnessy, *Civil Engineering*, August, 1941, p. 491.

(25) **"Hydraulic Tests on the Spillway of the Madden Dam,"** by R. R. Randolph, *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 1080.

(26) **"Study of Hydrotechnical Questions by Means of Laboratory Researches on Reduced Scale Models. Comparison of the Results of Such Researches with Those of Direct Observation of the Natural Phenomena, with a View to Ascertaining How Far the Law of Similitude Is True,"** by Rudolf Seifert, *Paper No. 53*, 3d Communication, 15th International Cong. of Navigation, Venice, 1931.

(27) **"Hydraulic Laboratory Results and Their Verification in Nature,"** by H. D. Vogel, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 597.

(28) **"Annual Report of the Work of the Central Board of Irrigation, India, 1936–37,"** Simla, April 1, 1938, p. 67.

(29) **"Central Board of Irrigation Report of Proceedings of Research Commission,"** India, 1939, pp. 32 *et seq.*

(30) **"Model and Full Size Surge Tests on Large Branched Tunnel System of Detroit Water Supply,"** by A. C. Michael, *Journal*, A. W. W. A., Vol. 28, March, 1936, p. 295.

(31) **"Performance of Prototype During Floods of October and December 1937 Compared with Flow Conditions Observed in the Models; Supplemental Report—Hydraulic Model Tests—Spillway and Outlet Works, Tygart River, Reservoir Dam, Grafton, W. Va.,"** U. S. Engr. Office, Pittsburgh, Pa., April 21, 1938.

(32) **"Report on Investigation of Artificial Stream-Control Structures,"** National Bureau of Standards, Washington, D. C., May 1, 1939, p. 13.

(33) **The Experiment Station Bulletin (Hydraulics),** U. S. Waterways Experiment Station, Vicksburg, Miss., Vol. 2, No. 3, July 1, 1939, pp. 5 and 17; Vol. 3, No. 1, February 1, 1940, pp. 1–6; Vol. 3, No. 2, June 15, 1940, pp. 1–6.

(34) **"Model Studies of Penstocks and Outlet Works,"** *Boulder Canyon Project Final Reports*, Bulletin 2, Pt. IV, Hydraulic Investigations, Denver, Colo., U. S. Dept. of the Interior, Bureau of Reclamation, 1938.

(35) **"Comparison of Field and Model Hydraulic Experiments, Boulder Outlet Works,"** *Technical Memorandum No. 610*, Bureau of Reclamation, U. S. Dept. of the Interior, Denver, Colo., July 22, 1940.

(36) **"Experimental and Practical Experience with the Gibson Method of Water Measurement,"** by N. R. Gibson and E. B. Strowger, *Transactions*, A. S. M. E., Vol. 57, 1935, p. 213.

(37) **"Modern Developments in Fluid Dynamics,"** by S. Goldstein, Clarendon Press, Oxford, Vol. II, 1938, p. 360.

(38) **"Loss in Oblique-Angled Pipe Branches,"** by D. Thoma, *Bulletin 3, Transactions of the Hydraulic Institute of the Munich Technical Institute*, A. S. M. E., New York, N. Y., 1935.

(39) **"Hydraulic Model Experiments for the Design of the Norris Dam,"** *Technical Memorandum No. 406*, Bureau of Reclamation. (*a*) p. 121.

(40) **"Models Cut Costs and Speed Construction,"** by J. C. Stevens, *Civil Engineering*, October, 1936, p. 674.

(41) **"Report of the Board of Engineers on Deep Waterways,"** Pt. I, 1900, p. 291.

(42) **"Weir Experiments, Coefficients, and Formulas,"** by R. E. Horton, *Water Supply and Irrigation Paper No. 200*, U. S. Geological Survey, 1907. (Reference to formulas and experiments of Bazin, Dubuat, Francis, Fteley and Stearns, Nelles, and others may be found in this paper.)

(43) **"Experiments on Small Weirs and Measuring Modules,"** by J. C. Stevens, *Engineering News*, Vol. 64, No. 7, August 18, 1910, p. 171.

(44) **"Report on Weir in the Niagara River,"** *Engineering Record*, Vol. 68, No. 8, p. 206. (Digest of a report of the International Waterways Commission, June 20, 1913. Digest includes all significant data and is more generally available.)

(45) **"The Submerged Weir as a Measuring Device,"** by G. N. Cox, *Bulletin No. 67*, Univ. of Wisconsin Eng. Experiment Station, Madison, Wis., 1928.

(46) **"Flow of Flood Water Over Railway and Highway Embankments,"** by D. L. Yarnell and F. A. Nagler, *Public Roads*, Vol. 11, No. 2, 1930, p. 30

(47) **"Tests of Broad-Crested Weirs,"** by James G. Woodburn, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 387.

(48) **"Supplemental Tests—Weirs with Aprons Inclined Up Stream and Down Stream,"** by A. R. Webb, *ibid.*, p. 408.

(49) **"Recreation Dam Built in Miami Reservoir,"** by C. H. Eiffert, *Engineering News-Record*, Vol. 120, No. 4, January 27, 1938, p. 137.

(50) **"Submerged Weir Formula Verified,"** by G. N. Cox, *ibid.*, Vol. 123, No. 9, August 31, 1939, p. 52.

(51) **"Concrete Controls for Stream-Gaging Stations,"** by O. W. Hartwell, *Civil Engineering*, February, 1939, p. 121.

(52) **"Hydraulic Studies of a Model of the University Dam at Iowa City,"** by C. L. Morgan, June, 1938 (graduate thesis, Dept. of Mechanics and Hydraulics, State Univ. of Iowa).

(53) **"Model Study of Plans for Channel Improvement at Grand Tower Reach, Mississippi River,"** *Technical Memorandum No. 114-1*, U. S. Waterways Experiment Station, Vicksburg, Miss., August 10, 1937.

(54) **The Experiment Station Bulletin (Hydraulics)**, U. S. Waterways Experiment Station, Vicksburg, Miss., Vol. 1, No. 2, October 1, 1938, pp. 5–6.

(55) **"Translatory Waves in Open Channels,"** by Horace W. King, *Civil Engineering*, June, 1933, pp. 319–321.

(56) **"Flow of Water in Tidal Canals,"** by Earl I. Brown, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), pp. 749–834.

(57) **"Fluid Mechanics for Hydraulic Engineers,"** by H. Rouse, *Engineering Societies Monograph*, 1938, pp. 353–402.

II. Notation

The following symbols, used in the Symposium, conform essentially with "Letter Symbols for Hydraulics," prepared by a Committee of the American Standards Association with Society representation and approved by the Association in 1942.[14]

A = area of cross section, a subscript denoting the section location:
- A_c = area of culverts;
- A_l = area of lock chamber;

b = length of weir crest;

C = discharge coefficient:
- C_l = lock coefficient;
- C_m = coefficient determined by model tests;
- C_q = coefficient of discharge;

D = diameter;

f = friction coefficient, subscripts m and p referring to model and prototype, respectively;

g = acceleration due to gravity;

H = head above a given datum plane: H_t = measured tailwater elevation, referred to the elevation of the crest;

[14] ASA—Z10.2—1942.

h = pressure heads at selected sections indicated by appropriate subscripts:

h_e = drop in pressure at entrance;

h_f = loss of head due to friction;

h_p = pressure head differential between two sections;

h_v = head, velocity;

h_x = pressure at any piezometer in the bellmouth;

K = a discharge coefficient defined by Eq. 11; also a constant in Eq. 3;

K_o = observed constant in the rating equation (Table 2) for a flow meter;

K_s = Scobey's friction coefficient;

K_t = theoretical constant in the rating equation (Table 2) for a flow meter;

L = lengths, such as center-line distances from the bellmouth, with subscripts to denote section locations; the distance required to produce normal distribution below an intake in a smooth pipe:

L_r = geometric scale ratio, $\frac{L_p}{L_m}$;

L_x = a length factor;

n = constant exponent defined by Eq. 11;

p = pressure, in pounds per square foot; $\frac{p}{w}$ = pressure head, feet of water;

Q = time rate of discharge, subscripts m and p denoting model and prototype, respectively;

q = discharge per unit width;

$\mathbf{R}$ = Reynolds' number;

r = radius;

V = average velocity at a given cross section;

V_l = lock-chamber rate of rise or fall;

$\mathbf{W}$ = Weber number;

w = specific weight of water in pounds per cubic foot; $\frac{p}{w}$ = pressure head, feet of water;

y = depth of flow;

Δ = central angle.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

DISTRIBUTION GRAPHS OF SUSPENDED-MATTER CONCENTRATION

By Joe W. Johnson,[1] Assoc. M. Am. Soc. C. E.

Synopsis

Damages or benefits resulting from sedimentation at various points in a drainage basin may be due to different types of sediment, possibly coming from different sources. For instance, flood and drainage damage caused by stream-channel aggradation appears to be primarily a bed-load problem, involving the deposition of relatively coarse material. On the other hand, fine material is responsible for much of the deposition in irrigation and drainage canals and the increased cost of water purification in many public water-supply systems. Both fine and coarse material contribute to the deposits in reservoirs, but the finer grade of sediment normally is the chief source of damage, because it constitutes the greater portion of the load. Because different types of damage are caused in different proportions by the two classes of sediment, it is important in sediment-load investigations that, not only the total load of the stream be determined, but also the relative proportions of fine and coarse material. These are the subjects discussed in the paper.

General

With the exception of dissolved matter, the total solids load passing a particular cross section in a stream may be classified either as bed-material load or wash load, with a definite grain size representing the division between the two classes (3).[2] The bed-material load, or "bed load" for brevity, is that part of the total solids composed of the relatively coarse material (coarser than the division grain size), and it moves by rolling or sliding along the bed, or by long jumps between points of contact with the bed while in temporary suspension, at a rate related to the stream discharge. The wash load, except

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **March 1, 1942.**

[1] Asst. Prof., Dept. of Mech. Eng., Univ. of California, Berkeley, Calif.; formerly, Hydr. Engr., SCS, Washington, D. C.

[2] Numerals in parentheses, thus (3), refer to corresponding items in the Bibliography, in the Appendix.

for a negligible percentage, moves in suspension at a rate that bears no direct relation to discharge and is composed of the relatively fine material (finer than the dividing grain size). The wash load constitutes much the greater part of the material in a suspended-load sample taken at flood stage, and may be regarded as sediment in relatively permanent suspension.

The particular grain size that divides the sediment load into these two classes varies in different streams and, generally, throughout the length of a particular stream, although it has a definite value in a particular reach of a stream. For example, the dividing grain size in the Colorado River near Imperial Dam is 0.01 mm (fine silt) (10), whereas in the Enoree River near Greenville, S. C., the size is 0.351 mm (medium sand) (3). The dividing grain size can be determined from mechanical analysis curves of samples taken from the stream bed immediately after a flood, and is the finest grain size existing in appreciable quantities.

The bed material (coarse material) in a stream system ordinarily changes slowly in composition over a long period of years, if at all, and the quantity moved is mainly in response to discharge. The quantity of wash material (fine material), however, may vary greatly from year to year and from season to season, because of the changing conditions of supply that are independent of conditions of flow. The most important variables of these completely interrelated conditions of supply are vegetal cover, tillage methods, stage of soil erosion, and rainfall characteristics.

Observations made at the Soil Conservation Service (SCS) sediment-load laboratory near Greenville (2) have shown the relative magnitudes of the two types of sediment load in a representative Piedmont stream. Of the total sediment load passing the station in the Enoree River during its two years of operation, more than 90% was found to consist of wash material.

Although this ratio between the annual amounts of the two classes of sediment load is not expected to be the same for all streams, or even constant for the Enoree River, it probably represents fairly well the general order of magnitude for streams of this size in the humid sections of the United States where the amount of wash material subject to erosional attack is essentially unlimited.

Suspended Load

Vertical Distribution.—A general theory of the vertical distribution of suspended sediment in a turbulent stream of water has been established as a result of extensive investigations on the transportation of sediment, in both the United States and Europe. Based upon this theory, the total suspended-sediment load transported in a vertical section can be estimated from a determination of the concentration and the particle size distribution of the sediment at a single point in the vertical, and the hydraulic characteristics of the stream. In actual practice, however, the procedure is more simple because it is necessary to apply the theory only to sediment coarser than 0.075 mm (200-mesh), since material finer than this size appears to be uniformly distributed throughout the section in most streams, and a single accurate sample taken at any point in the stream gives the concentration applicable to the entire cross section.

Sampling and Analysis.—By accurate sampling and grade-size analysis of the suspended matter in a stream, it is possible to compute the total load of coarse material—material of sizes found in appreciable quantity in bed deposits —carried in temporary suspension; and, by making a sufficiently large number of observations over a wide range in stage, a relationship between coarse load and discharge can be established for a particular location. Estimates of the future suspended load of coarse material, therefore, can be made from this established relationship, and from a record of the discharge, without the necessity of further sampling. The load of fine material, on the other hand, can be estimated accurately merely from a dip sample. However, it bears no direct relationship to the discharge, and in the past continuous sampling has been necessary to estimate the total volume of fine material carried in a stream during a flood.

Observation Periods.—In small flashy streams almost all of the annual load of wash material is carried during a few days out of the year (6), and the content of wash material may vary many 100% within the period of an hour during the rising and falling stage of the stream. To the present time, no method of wide applicability, except frequent sampling, has been developed for estimating or computing, with even approximate reliability, the total suspended load carried by any small stream. It is true that on some large rivers an approximate relationship (1) has been developed between discharge and suspended load—presumably including both fine and coarse material—and, from this, estimates of total sediment transportation have been made. However, where such relationships have been developed, the period of observation, in general, has extended over only one or two years when conditions of supply apparently were approximately constant. Continued observations over a longer period of time usually have resulted in an increased scattering of data (9). The discussion to follow is concerned primarily with the transportation of the fine material in small streams in humid climates and gives a general description of some of the sources and factors controlling the supply of material.

Sources of Sediment.—The load of fine material in a stream appears to have its main source, not in the stream bed, but in other parts of the drainage basin. It is possible, of course, that a small amount of fine material in any particular flood may come from the stream bed, where it was deposited in pools during low stage, or where it became trapped between larger bed particles, and was released when the bed material started to move during a higher stage. In general, however, the major sources of sediment are cultivated lands, road cuts, roadside ditches, gullies, and caving stream banks. The amount of material that may enter a stream from these sources during a particular storm depends upon such complex, interrelated, and, in part, changing factors as topography, soils, shape and size of the drainage basin, land use, vegetal cover, and surface and underground storage, as well as upon rainfall intensity and duration. In any drainage system, however, certain of these factors may be considered constant, whereas others have a seasonal variation.

Results of Past Studies.—Numerous investigations of soil loss and runoff from relatively small agricultural plots give general information on the prin-

ciples underlying the supply of erosional debris. These studies have shown, in general, that the sediment concentration in runoff water from a plot usually increases to a maximum within a very short time, and then decreases as the duration of application of water continues, approaching a constant minimum value asymptotically. The high initial sediment concentration appears to be the direct result of rain impact. However, rain packing and adhesion from wetting and the swelling of colloids, as the rainfall continues, as well as the variation of soil porosity with depth, appear to be the main factors tending to reduce concentration. The variation of the infiltration capacity with duration of rainfall also has been observed to follow a law similar to that of sediment concentration.

Erosion Principles.—Erosion by rain impact and overland flow applies more specifically to the land surface and the exposed side slopes of road cuts, ditches, and gullies, but once water is concentrated in relatively large, well-defined channels, the principles of stream erosion more nearly apply. Stream erosion probably is closely related to discharge, because, as a rule, the greater the discharge, the greater will be the velocity, and hence the capacity to dislodge and transport particles. In any event, however, and despite the complicated mechanics by which material is placed in suspension and transported to the outlet of the ordinary small plot, the concentration of fine material appears, in general, to vary with the surface runoff. The same reasoning should apply to a natural stream; although, obviously, the problem is more complicated, because rainfall distribution and areas of sediment production, which are in a sense a system of small plots, vary considerably in their characteristics throughout the drainage basin.

Hydrographs and Distribution Graphs.—It has been observed generally that, following a rainfall during which surface runoff occurs, the discharge of this surface runoff into stream channels simultaneously over a drainage basin results in pronounced rises in stream levels, followed by periods of decline. The concentrations of suspended matter in streams also have been observed to rise and decline in a similar manner (Fig. 1).

The plotted graph of stream flow, showing the rises and declines of runoff, has been used by many investigators as a basis for studying the phenomenon of surface runoff. L. K. Sherman, M. Am. Soc. C. E. (8), conceived the idea that, for a particular drainage area, the surface runoff from rainfall occurring within the same time interval will produce hydrographs, the ordinates of which will vary with the amount of the surface runoff. Mr. Sherman has given the term "unit graph" to the hydrograph of surface runoff that results from rain falling within a unit of time, as a day or an hour. If the unit hydrograph of surface runoff is modified to show the proportional relations of its ordinates in the percentage of total surface runoff, the resulting graph is termed a "distribution graph." The unit-hydrograph method is based upon the hypothesis that in a particular drainage basin surface runoff from rainfall occurring in a unit of time will produce hydrographs of approximately equal bases, and the ordinates will vary with the intensity of the net rainfall.

From the consideration, as suggested by plot studies, that surface runoff and suspended-matter concentration appear to be related to the same factors,

an investigation was made as part of the SCS sediment-load studies to determine if any such relationship actually existed in a natural stream. Data from the East Fork of Deep River, near High Point, were used for this study, and

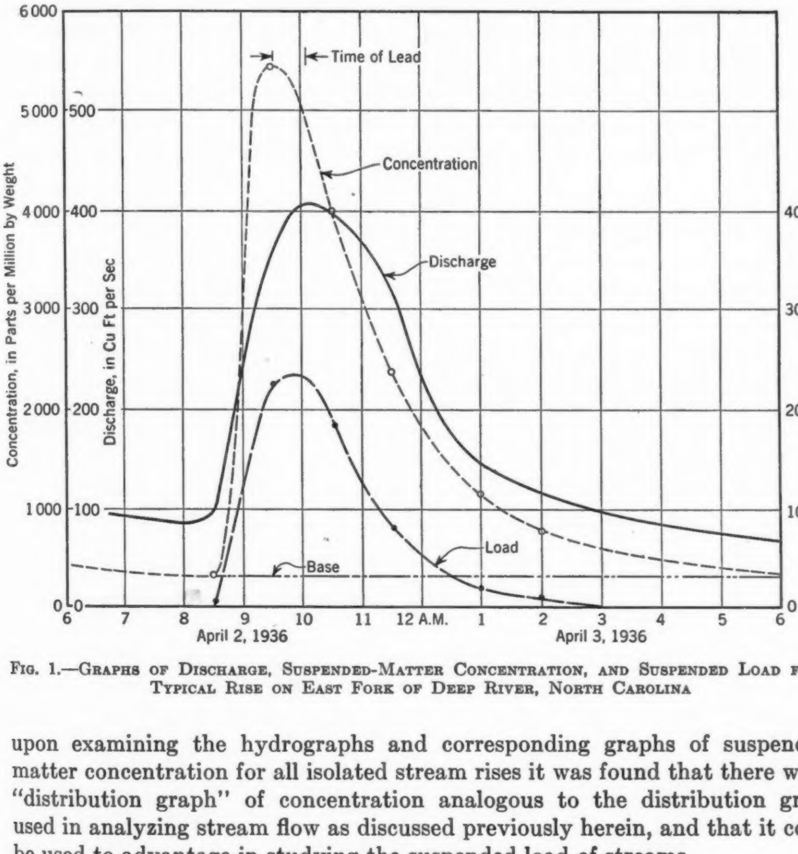


FIG. 1.—GRAPHS OF DISCHARGE, SUSPENDED-MATTER CONCENTRATION, AND SUSPENDED LOAD FOR A TYPICAL RISE ON EAST FORK OF DEEP RIVER, NORTH CAROLINA

upon examining the hydrographs and corresponding graphs of suspended-matter concentration for all isolated stream rises it was found that there was a "distribution graph" of concentration analogous to the distribution graph used in analyzing stream flow as discussed previously herein, and that it could be used to advantage in studying the suspended load of streams.

DEVELOPMENT OF DISTRIBUTION GRAPH

The data used in this study were obtained on the East Fork of Deep River during an extensive suspended-matter sampling program conducted by the United States Geological Survey in cooperation with the SCS and the North Carolina Agricultural Experiment Station. Collection of water samples for the determination of sediment concentration was made once a day during periods of low water, and at more frequent intervals during periods of storm runoff. Fig. 1 illustrates a typical example of a stream rise, and shows the usual frequency of sampling considered necessary to obtain sufficient information for the determination of variations in content of suspended matter. Other basic data secured included the stream-discharge and precipitation records

from three standard and three recording gages located in and about the 13.9-sq-mile drainage basin.

The basic data from the gaging station are sufficiently complete to give valuable general information on the movement of suspended matter in a typical stream in the Piedmont. The concentration of suspended matter, as recorded and used in the following analysis, includes both fine and coarse material. A few mechanical analyses of samples taken at flood stages, however, showed that the amount of coarse material in suspension was insignificant as compared to the fine material that comprised most of the wash load, and could be disregarded in studies where the movement of fine material was of primary interest.

It should be noted here that, if the movement of the coarse material is of primary importance, careful sampling and special equipment are necessary to obtain an accurate estimate of the total load of coarse material. This was evident in the Enoree River studies, where it was found that the coarse material in suspension may be an appreciable percentage of the total load of coarse material, even though it is small in comparison with the load of fine material.

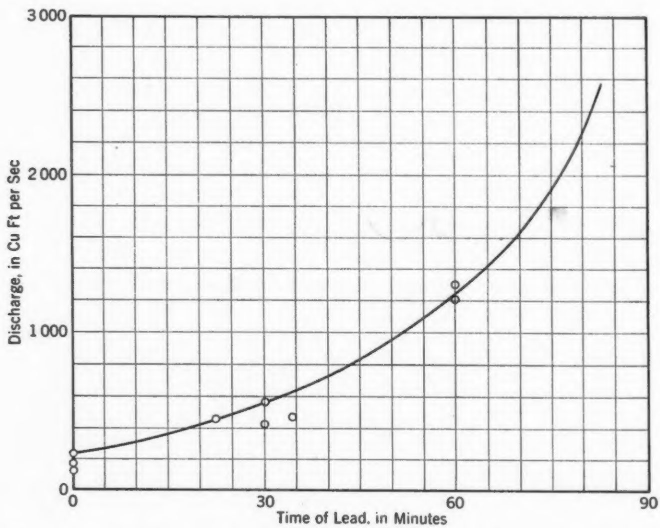


FIG. 2.—RELATION BETWEEN MAXIMUM DISCHARGE AND THE TIME THAT MAXIMUM SUSPENDED-MATTER CONCENTRATION PRECEDES THE MAXIMUM DISCHARGE

Fig. 1 illustrates the variation of concentration during a typical isolated rise, and shows the general conditions under which suspended-matter content increases very rapidly from the beginning of the rise until about the peak stage. The maximum content either occurs somewhat in advance of, or coincides with, the maximum discharge of the stream, depending upon the stream discharge (Fig. 2). The intervening period of time, between the occurrence of the peak concentration and the occurrence at the peak discharge, is termed the "time-of-lead."

After the peak concentration has been reached, the content of suspended matter decreases with the falling stage of the stream, but the decrease usually is at a less rapid rate than the increase during the rising stage. In general, the recession curve of concentration has a long "tail," as evidenced by the muddy appearance of a stream for a considerable time after discharge has returned to approximately normal flow. As the discharge and suspended-matter concentration continue to decrease, a point is reached where the sediment load becomes negligible in comparison with that transported during the rise. In a small stream this condition usually occurs only a relatively few hours after the crest has passed. This is illustrated by the load graph in Fig. 1, which shows that only a negligible load was transported after 3:00 a.m. on April 3, 1936.

A distribution graph is calculated as follows: For a particular stream rise in which a sufficient number of water samples have been taken to define the variation in concentration throughout the rise, curves of suspended-matter concentration, discharge, and suspended load are plotted on a single diagram as in Fig. 1. To segregate the concentration due wholly to the rise in question, a horizontal base line is drawn at an ordinate, which appears, by inspection, to be the value of the concentration prior to the rise. (The use of a recession curve, as in stream-flow studies, adds little to the accuracy of the results.) Having separated the base concentration from the observed values, the unit graph of concentration is represented by the ordinates between the base line and the actual graph of concentration. The time base of the concentration graph is considered arbitrarily to extend from the beginning of rise (8:30 p.m., Fig. 1) to the time when the concentration has receded to the point beyond which the load can be considered negligible (3:00 a.m., Fig. 1). The base of the unit graph is divided arbitrarily into time intervals, and the average concentration for each interval is then determined. These values are totaled and the percentage of the total occurring during each of the periods is computed. Time intervals of 30 min and 1 hr were found suitable for representing accurately the shape of the graphs for rises occurring in the East Fork of Deep River.

Distribution graphs were prepared for all the isolated rises that occurred in the East Fork of Deep River during the four years of record from April, 1934, to June, 1938. After reducing these various graphs to a constant time interval, it was found that the shape of the graphs appeared to be a function of the period of stream rise, and somewhat of the intensity of the rainfall. The period of rise in this case is defined as the time intervening between the beginning of surface runoff and the occurrence of the peak discharge. Fig. 3 shows the distribution graphs for ten storms in which the period of rise was 2 hr or less.

These various graphs were prepared from rises that occurred during all seasons of the year, and, consequently, represent various conditions of land use, vegetal-cover, soil-moisture, and rainfall characteristics. Rises with the highest runoff and soil loss generally occurred during April and September. In September the losses appeared to be the result of torrential rains, whereas in April a high initial soil-moisture content was the most important factor. Two rises occurred during July as a result of high-intensity rains, but the runoff and subsequent erosion were not as severe as for April and September, probably because of the dry and porous condition of the soil, but perhaps also because

the total rainfall was less and the evaporation and transpiration probably were greater.

The average distribution graph of the superimposed graphs in Fig. 3 is tabulated and plotted in Fig. 4, together with the average distribution graphs

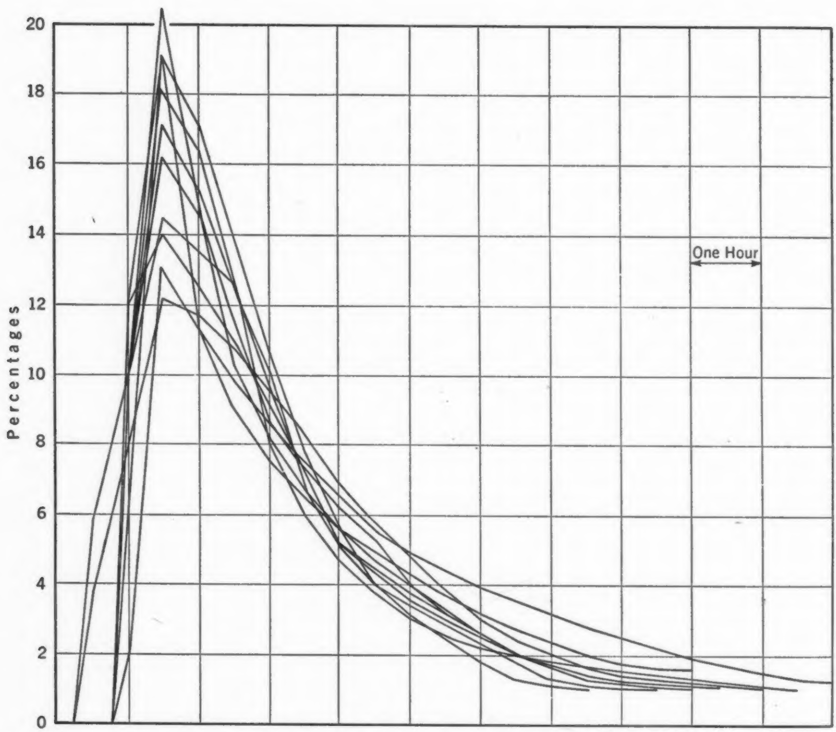
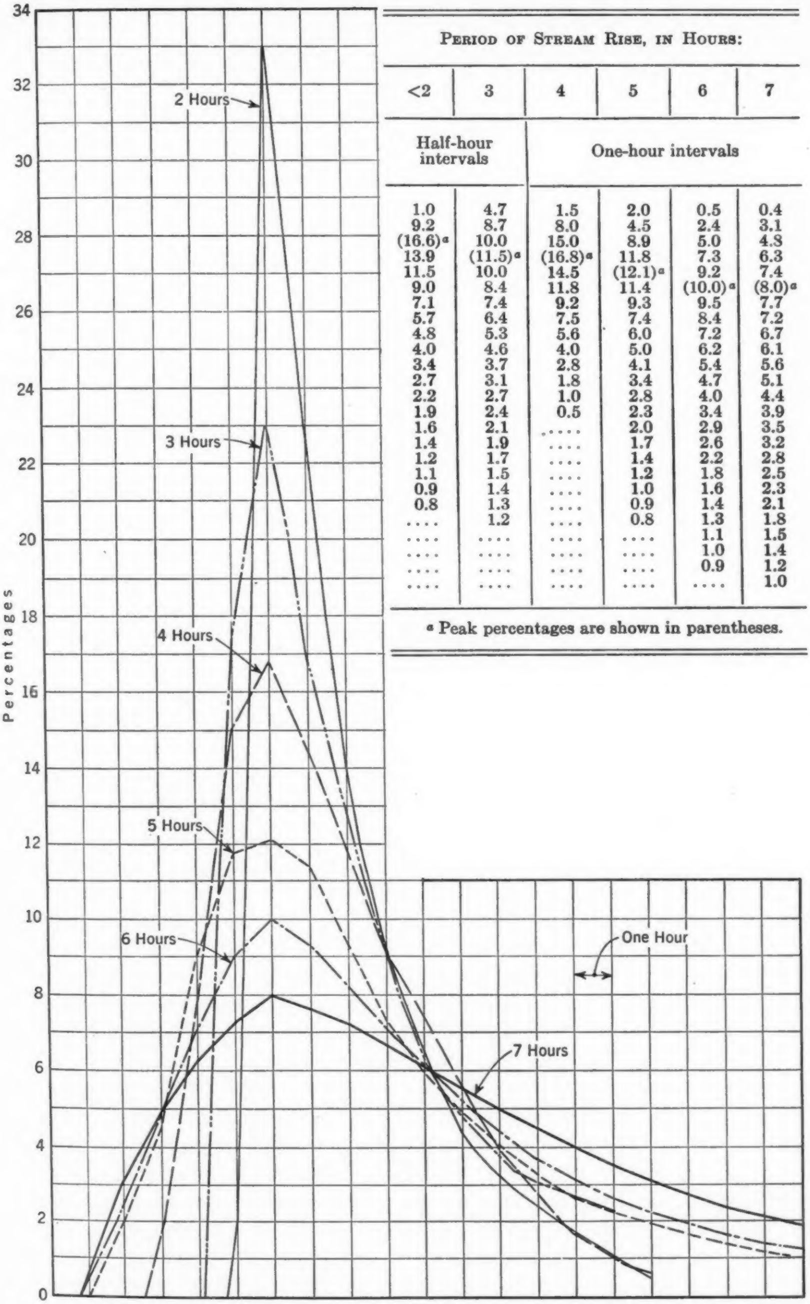


Fig. 3.—Superimposed Distribution Graphs of Suspended-Matter Concentration for Ten Rises, on the East Fork of Deep River, in Which the Period of Stream Rise Was 2 Hr or Less

for other periods of rise, the percentages in all cases being expressed for a time interval of 1 hr. The graph for a period of rise of 2 hr or less is characteristic of high-intensity rains of short duration, when rain impact probably is responsible for placing most of the sediment into suspension. The graphs for the longer periods of rise are characteristic of long duration, low-intensity rains, where rain impact probably is of secondary importance in placing sediment in suspension.

Rises resulting from uniformly distributed rainfall should be used, where possible, in preparing a set of distribution graphs. Although rainfall records are not necessary in the application of the graphs, such information is of importance in judging the relative reliability of the computations as affected by rainfall distribution.



Period of Stream Rise, in Hours:					
<2	3	4	5	6	7
Half-hour intervals		One-hour intervals			
1.0	4.7	1.5	2.0	0.5	0.4
9.2	8.7	8.0	4.5	2.4	3.1
(16.6)[a]	10.0	15.0	8.9	5.0	4.8
13.9	(11.5)[a]	(16.8)[a]	11.8	7.3	6.3
11.5	10.0	14.5	(12.1)[a]	9.2	7.4
9.0	8.4	11.8	11.4	(10.0)[a]	(8.0)[a]
7.1	7.4	9.2	9.3	9.5	7.7
5.7	6.4	7.5	7.4	8.4	7.2
4.8	5.3	5.6	6.0	7.2	6.7
4.0	4.6	4.0	5.0	6.2	6.1
3.4	3.7	2.8	4.1	5.4	5.6
2.7	3.1	1.8	3.4	4.7	5.1
2.2	2.7	1.0	2.8	4.0	4.4
1.9	2.4	0.5	2.3	3.4	3.9
1.6	2.1	....	2.0	2.9	3.5
1.4	1.9	....	1.7	2.6	3.2
1.2	1.7	....	1.4	2.2	2.8
1.1	1.5	....	1.2	1.8	2.5
0.9	1.4	....	1.0	1.6	2.3
0.8	1.3	....	0.9	1.4	2.1
....	1.2	....	0.8	1.3	1.8
....	....	....	....	1.1	1.5
....	....	....	....	1.0	1.4
....	....	....	....	0.9	1.2
....	....	....	....	....	1.0

[a] Peak percentages are shown in parentheses.

Fig. 4.—Average Distribution Graphs of Suspended-Matter Concentration, in East Fork of Deep River, for Various Periods of Stream Rise

Application of the Distribution Graph

To illustrate the application of the distribution graph of suspended-matter concentration in the calculation of suspended load, data were used from a rise that occurred in the East Fork of Deep River on November 7, 1935. Fig. 5(*b*) shows the observed discharge and suspended-matter concentration resulting from a rain of the characteristics illustrated by the chart shown in Fig. 5(*a*). The record from only one of the recording rain gages is shown. This is a tipping-bucket recording gage maintained by the United States Weather Bureau at the Greensboro, N. C., airport.

The use of the distribution graph, as developed, requires that the stream hydrograph and one measurement of suspended-matter concentration during a rise be available. If the hydrograph is composed of several peaks, an observed sediment concentration for each peak is required. It is desirable that the sample for concentration be taken at, or shortly after, the peak discharge, because, during the rising stage, the concentration often fluctuates erratically; whereas, during the falling stage, the rate of change in suspended-matter content decreases uniformly, and at a less rapid rate.

The hydrograph shown in Fig. 5(*b*) consists of two peaks, and therefore two observed concentrations are used in applying the distribution graph. With a known value of concentration and a selected distribution graph oriented with respect to the peak discharge according to Fig. 2, a computed distribution of concentration can be made for each peak. Where the effect of two peaks overlaps, the individual curves are calculated separately, and then added graphically to give the graph of suspended-matter concentration for the complete hydrograph. The suspended load then is easily calculated from the discharge and the calculated concentration. If the load is calculated for several points throughout a rise, a complete graph of the load may be constructed. The total tonnage of sediment is represented by the area under the load curve. If desired, this load then may be converted to volume by selection of a volume-weight ratio.

In the first rise, shown in Fig. 5(*b*), the period of rise is approximately 4 hr. With a peak discharge of 70 cu ft per sec, the time-of-lead is zero according to Fig. 2, and the peak concentration therefore is assumed to coincide with the peak discharge. The 4-hr distribution graph, shown in Col. 3 of the tabulation in Fig. 4, is used and plotted in Fig. 5(*c*), with the peak coinciding with the time of peak discharge. It so happens that the observed concentration of 860 ppm, as selected for use in the calculations, was taken at the instant of peak discharge; consequently, it is the concentration at which the curves of calculated and observed concentration will arbitrarily be made to coincide. By multiplying this peak concentration of 860 ppm by the ratio of the ordinate percentages for each time interval of the distribution graph to the peak percentage, a calculated curve (curve a, Fig. 5(*c*)) of concentration is obtained for the first rise. It may be noted that the recession part of this curve extends over a period of time sufficiently long to contribute to the total amount of sediment in suspension during the second rise; consequently, to distribute the concentration in the second rise, a segregation of the concentration resulting from the first rise will be required.

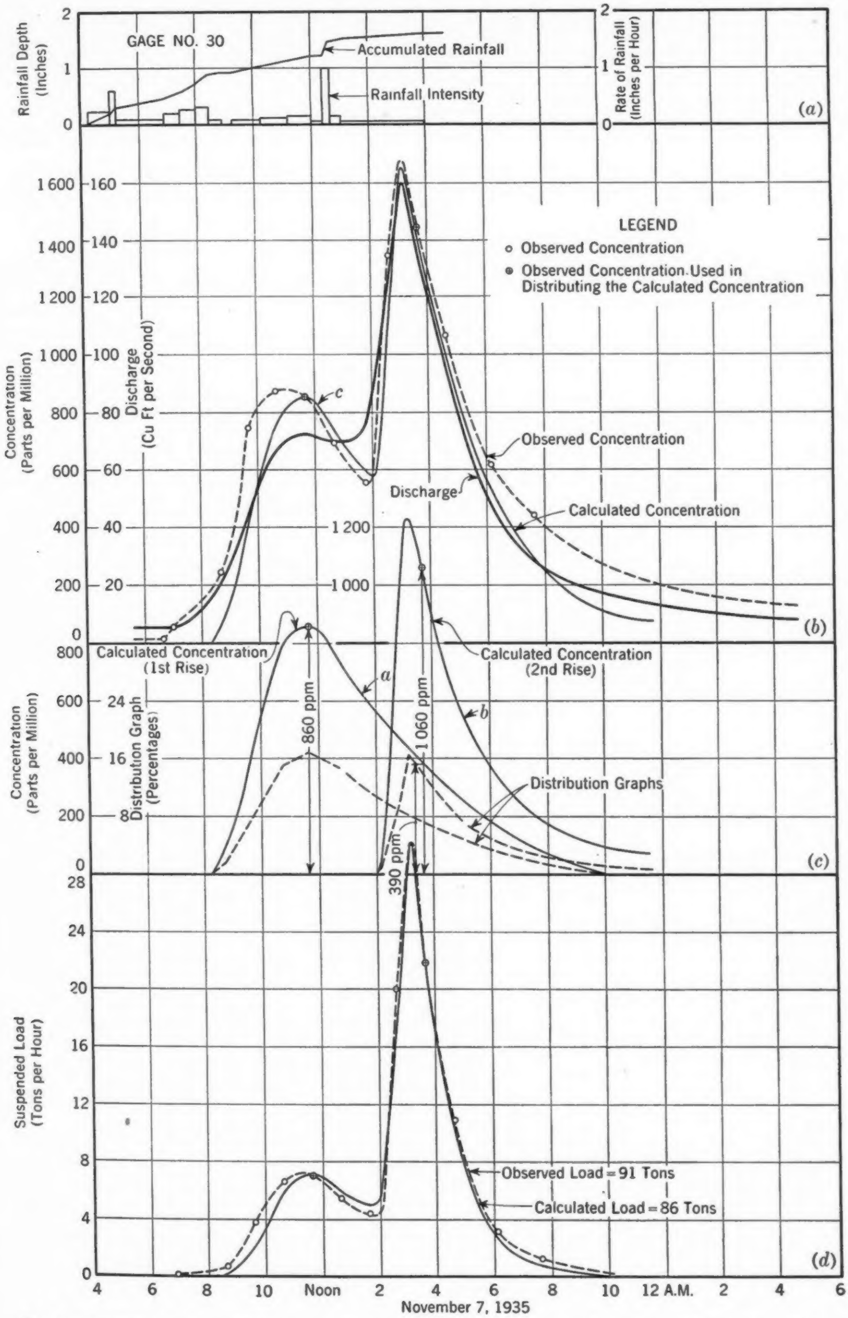


Fig. 5.—Illustration of Method of Calculating the Suspended Load of a Stream by Use of Distribution Graphs of Suspended-Matter Concentration

From the known period of rise and the peak discharge, the 2-hr distribution graph (Col. 1, Fig. 4) is selected and oriented (Fig. 5(*c*)), so that the peak percentage coincides with the peak discharge, because here again Fig. 2 shows a zero time-of-lead for this discharge. The observed concentration to be used in calculating the curve of concentration in this second rise is segregated from the first rise by subtracting the calculated concentration of the first rise for the corresponding time; that is, a concentration of 390 ppm (read from the calculated concentration curve for the first rise) is subtracted from the total observed concentration of 1,450 ppm to give a segregated concentration of 1,060 ppm. The distribution-graph ordinate for this corresponding time is 13.9%. By multiplying the segregated concentration of 1,060 ppm by the ratio of the distribution-graph percentages (Col. 1, Fig. 4) and the percentage of 13.9, a curve of concentration (curve b, Fig. 5(*c*)) is obtained for the second rise. By adding graphically the individual curves for each rise (curves a and b, Fig. 5(*c*)) a calculated curve of suspended-matter concentration is obtained for the complete hydrograph.

Although these calculated and observed curves of concentration were made arbitrarily to coincide at two points, visual examination shows that agreement between the two can be considered as satisfactory. Close agreement is further confirmed by the load curves in Fig. 5(*d*) where the calculated load for the flood period agrees to within 5% with the observed load. Note that the suspended load transported during periods of low water is negligible in comparison with that transported during floods; hence, if estimates of load are made only for flood periods, sufficiently precise estimates will be obtained for most practical problems.

The primary advantage of distribution graphs is the fact that they permit an estimate of the suspended load of a stream to be made with only a single observation of suspended-matter concentration during a rise. This should be of particular value in localities where sufficient funds are not available for obtaining personnel for sampling throughout the flood period. Fewer samples for analysis also decrease the laboratory costs. This method, of course, is subject to all limitations applicable to the use of the "unit graph" in hydrologic studies; that is, sufficient records at a particular location on a stream must be available, or some reasonable assumptions must be made to obtain a reliable set of distribution graphs.

The application of this method to a compound hydrograph in the East Fork of Deep River is shown in Fig. 6. For this flood the time-of-lead, the period of rise, and the observed concentration used in distributing the concentration are shown for each rise. The actual recorded peak discharge was used in orienting the position of the distribution graph with respect to the peak discharge by the use of Fig. 2. In some instances, however, it might be more desirable to use the peak discharge after segregation from the antecedent flow. In general, however, it appears that considerable leeway is permissible in the selection of a distribution graph and time-of-lead, without greatly influencing the estimate of the suspended load. It is to be noted that the calculated and observed loads for the flood period of July 29–31, 1936 (Fig. 6), agree within 10%, an accuracy which, for most practical purposes, may be considered as satisfactory.

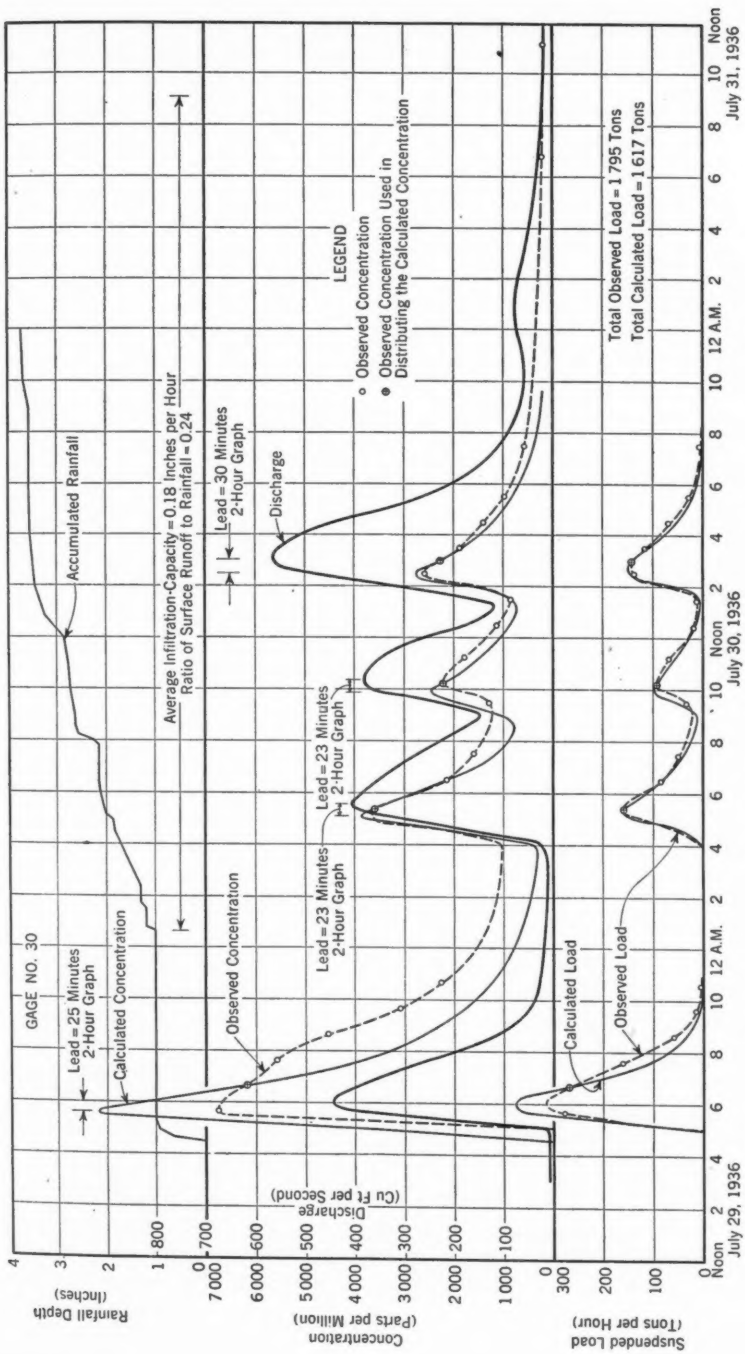


FIG. 6.—EXAMPLE OF THE APPLICATION OF THE DISTRIBUTION-GRAPH METHOD TO THE CALCULATION OF SUSPENDED LOAD IN A COMPOUND HYDROGRAPH ON EAST FORK OF DEEP RIVER

Items that may be of particular interest are the average infiltration capacity and the ratio of surface runoff to rainfall for the stream rise, shown in Fig. 6. On the graph of rainfall is indicated the portion of the rainfall that was assumed to contribute the segregated surface runoff used in the calculation of both infiltration capacity, and the ratio of surface runoff to rainfall. The average infiltration capacity for the watershed has been calculated by the method described by Robert E. Horton, M. Am. Soc. C. E. (4). Calculation of the average infiltration capacity for several other flood periods was made and, as would be expected, showed a rather wide range of values which varied not only with season but also with antecedent rainfall and condition of the soil.

Plot studies have shown that, for a constant slope, the main factors governing erosion are rainfall intensity, rainfall duration, initial moisture content of the soil, and the effect of the condition of the soil surface. The most important factor affecting runoff and soil erosion is the rainfall intensity, with the soil loss being affected to a greater degree. Describing the phenomenon briefly (7), when rain falls on a dry soil, the soil is slaked and thrown into suspension. If the rainfall intensity is high enough to cause runoff within a few minutes after the rain starts, this suspended material is carried off, and upon entering a stream becomes the major portion of the wash load. Rain falling on moist soil only packs it down and creates a pavement effect that sheds the water, but erosion is not as severe as on a soil originally in a dry state. If a rain continues to fall on a soil that originally was dry, most of the material readily available for transportion in suspension is soon washed away, and the remaining wet soil packs down into a smooth pavement-like surface. Although the runoff increases as the rain continues, the soil losses decrease.

That these general principles governing loss from small agricultural plots are also reflected in the suspended-matter content of a drainage stream has been confirmed by data obtained on the East Fork of Deep River on July 29–31, 1936 (Fig. 6). The first rise resulted from a high-intensity rain that fell on soil in a dry condition at a time of the year when the soil surface probably was in a pulverized condition. A relatively high suspended-matter concentration was attained as a result. Upon subsidence of this initial rise, additional rainfall occurred, with the resulting runoff and suspended-matter concentration apparently being decidedly affected by the first rise. As a result of rain packing, and the flushing away of most of the loose material during this initial rise, the suspended-matter concentration during the subsequent rises was considerably reduced. In fact, during the fourth and last rise in which the highest peak of discharge of the period was attained, the maximum concentration was but little greater than during the lowest of the peak discharges. This phenomenon of an apparent exhaustion of the supply of fine material, characterized by the suspended-matter concentration reaching an early peak, and then decreasing even though the discharge may continue to increase, also has been observed in the Enoree River investigations. Obviously, when the conditions governing the supply of fine sediment to a stream are changing almost continuously with time, a simple relationship between discharge and suspended load cannot be expected to exist.

The eventual solution of the problem of erosion will require considerable information on the rainfall characteristics during each storm. In reporting the results of 1940 studies on raindrops and erosion, J. O. Laws (5) made the following statement:

> "* * * intensity and duration records by themselves do not constitute an adequate description of rain. To adequately describe rain so far as erosion and infiltration are concerned, it is necessary in addition to know the size and velocity of its drop. The relative significance of the drop characteristics depends upon a multitude of factors, such as cover, slope, and soil type, and has yet to be evaluated."

Conclusions

1. By using a distribution graph of suspended-matter concentration in connection with the observed hydrograph and a single observed suspended-matter concentration, the total suspended load of fine material transported during a particular stream rise can be calculated with reasonable accuracy. Previous records are necessary, of course, in developing a set of distribution graphs for a particular location.

2. Although the shape of the distribution graph depends on the period of rise and, to some extent, on the intensity of rainfall, unless considerable basic data are available to define the effect of intensity adequately, an average distribution graph for various periods of the rise should be used.

Acknowledgments

The basic data for the East Fork of the Deep River drainage basin were obtained by the Quality of Water Division of the United States Geological Survey in cooperation with the Soil Conservation Service and the North Carolina Agricultural Experiment Station.

APPENDIX

Bibliography

(1) **"A Rating-Curve Method for Determining Silt-Discharge of Streams,"** by F. B. Campbell and H. A. Bauder, *Transactions*, Am. Geophysical Union, Annual Meeting 21, Pt. 2, 1940, pp. 603–607.

(2) **"Studying Sediment Loads in Natural Streams,"** by Gilbert C. Dobson and Joe W. Johnson, *Civil Engineering*, February, 1940, pp. 93–96.

(3) **"A Distinction Between Bed-Load and Suspended Load in Natural Streams,"** by H. A. Einstein, A. G. Anderson, and J. W. Johnson, *Transactions*, Am. Geophysical Union, Annual Meeting 21, Pt. 2, 1940, pp. 628–633.

(4) **"Determination of Infiltration-Capacity for Large Drainage-Basins,"** by R. E. Horton, *ibid.*, Annual Meeting 18, Pt. 2, 1937, pp. 371–385.

(5) **"Recent Studies in Raindrops and Erosion,"** by J. O. Laws, *Agricultural Engineering*, Vol. 21, No. 11, 1940, pp. 431–433.

(6) **"Suspended Matter in Several Small Streams,"** by S. K. Love, *Transactions*, Am. Geophysical Union, Annual Meeting 17, Pt. 2, 1936, pp. 447–452.

(7) **"The Effect of the Degree of Slope and Rainfall Characteristics on Runoff and Soil Erosion,"** by J. H. Neal, *Research Bulletin No. 280*, Missouri Agricultural Experiment Station, 1938.

(8) **"Streamflow from Rainfall by Unit-Graph Method,"** by L. K. Sherman, *Engineering News-Record*, Vol. 108, No. 4, 1932, pp. 501–505.

(9) **"Missouri River,"** letter from the Secretary of War transmitting report from Chief of Engineers, H. R. Doc. No. 238, 73d Cong., 2d Session, 1935, p. 1182.

(10) **"Why Desilting Works for the All-American Canal?"** by C. P. Vetter, *Engineering News-Record*, Vol. 118, No. 8, 1937, pp. 321–326.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

STABILITY OF GRANULAR MATERIALS

Discussion

BY R. E. GLOVER, ESQ., AND F. E. CORNWELL, ESQ.

R. E. GLOVER,[14] ESQ., AND F. E. CORNWELL,[15] ESQ.[15a]—Professor Hennes objects to the use of Hooke's law for the material in the elastic zones, and quotes test results in support of his objection. The type of stress-strain relation expressed in Eq. 30 is obtained in a triaxial test when the second and third principal stresses are held constant and the first principal stress is raised progressively until failure occurs. The elastic modulus obtained by dividing the first principal stress by the total strain is variable, beginning at a finite value and descending toward zero as the test proceeds. The finite value at the beginning of the test is due to the initial pressure which must be placed on it when the test is set up. If the test is performed by starting all pressures at zero and thereafter bringing them all up together, so that a definite and stable relation is maintained among them, a different result will be obtained. In this case the modulus, obtained in the manner described, will start at zero and increase as the first principal stress increases.

A third result can be obtained by holding the second and third principal stresses constant, bringing the first principal stress about halfway to the failure point, and then determining the elastic modulus by observing the strains accompanying a partial release and restoration of the first principal stress. A change in density of the specimen due to a different packing will also modify the results. Some test values are given in Table 4.

In view of these differences it would appear that a closer approximation to the actual stress-strain relation than the simple one used by the authors could be chosen only after a careful study of the material, the proposed method of placing it in the dam, and the loads to be placed on it thereafter.

In connection with Figs. 8 and 9, Mr. Benscoter presents a discussion that appears reasonable for a granular mass with cohesion. The horizontal direct

NOTE.—This paper by R. E. Glover, Esq., and F. E. Cornwell, Esq., was published in November, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1941, by R. G. Hennes, Assoc. M. Am. Soc. C. E.; March, 1942, by Stanley U. Benscoter, Jun. Am. Soc. C. E.; and April, 1942, by Messrs. A. Hrennikoff, and D. P. Krynine.

[14] Senior Engr., Bureau of Reclamation, Denver, Colo.

[15] Asst. Engr., Bureau of Reclamation, Denver, Colo.

[15a] Received by the Secretary September 14, 1942.

stress at a point in Fig. 8 corresponding to point A, Fig. 9, although not a principal stress, is only slightly greater than a fourth of the vertical stress. Similarly, for point B, slightly below the surface, the direct stress parallel to the downstream slope is almost twice that perpendicular to the slope, although

TABLE 4.—THE ELASTIC MODULUS, COMPUTED FROM TESTS OF A FINE NATURAL SAND
(Grain Size, 0.04 to 0.60 mm)

CASE 1.—COMPUTED FROM VOLUME CHANGES PRODUCED BY THE APPLICATION OF HYDROSTATIC PRESSURE[a]			CASE 2.—COMPUTED FROM A TRIAXIAL TEST; INITIAL HYDROSTATIC PRESSURE, 100 LB PER SQ IN.[b]		
Test pressure (lb per sq in.)	Volume strain (in. per in.)	Elastic modulus (lb per sq ft)	Deviator stress (lb per sq in.)	Axial strain (in. per in.)	Elastic modulus (lb per sq ft)
10	0.02388	181,000	118	0.0105	1,618,000
25	0.02453	440,000	184	0.0215	1,232,000
50	0.02541	850,000	223	0.0323	994,000
100	0.02641	1,636,000	249	0.0435	824,000
....	....	....	267	0.0542	709,000

[a] Computed by dividing the stress by the total linear strain. The linear strain is computed by dividing the volume strain by 3. [b] Computed by dividing the deviator stress by the total axial strain after the start of the test.

these stresses are not the principal ones. As he states, Fig. 9 corresponds to non-linear solutions, whereas Fig. 8 was for linear ones.

Good agreement was found between predicted failure at the reentrant corner common to regions V, VI, and VII and actual performance of several models, when instability was caused by raising percolation gradients in region VII. These tests are discussed further herein under remarks on Professor Hrennikoff's discussion.

The writers agree with Mr. Benscoter that the application of mathematical theory to soil analysis problems is behind similar practice in other fields, and they look forward to further development. They also agree that new methods for measuring pressures in embankments are needed urgently.

Professor Hrennikoff comments at some length on the uniqueness of the solutions and concludes that the proposed solutions are unacceptable unless some additional proof of their reasonableness can be supplied. To support his conclusion he describes procedures by which he attempts to show that a wide variety of stress states could be installed in the dam by some artificial conditioning of the soil mass. The writers object to his demonstration on the following grounds: The passive, plastic-state, stress distribution requires the presence of shearing stresses on horizontal and vertical planes, and therefore it would be impossible to set up such a state by using a smooth wall in the manner he describes. This statement is true also for the active, plastic-state, stress distribution and all the possible intervening elastic states so long as the boundary slope is not horizontal. A similar difficulty will be encountered if the wall is to be made rough. In this case, any of the foregoing stress states could be set up because the shearing stresses required on the vertical planes could now be maintained; but it would not be possible to eliminate the central gap without altering the shearing stresses because a removal of the walls

would leave a discontinuity in the shear stresses along the plane of symmetry. This discontinuity represents a major violation of the requirement for equilibrium. If an elastic zone is introduced, these difficulties can be removed and the shear stress can be made to pass through zero at the plane of symmetry, as it should.

These difficulties with Professor Hrennikoff's demonstration are considered to be sufficiently serious to warrant a reconsideration of the adverse criticisms in his section 1. In the latter part of this section he objects to the use of plastic states in zones III and V of Fig. 4 and expresses a preference for an elastic state throughout for slopes as low as those shown. In reply to this objection it can be stated that a solution satisfying the elastic equations, and free from the aforementioned difficulties, will require the existence of stresses along the boundaries and these cannot be sustained by a cohesionless material. The active state was chosen for regions I and VII for the examples in Figs. 4 and 5 because it was considered to be the state which probably would exist in a granular mass that had been deposited by running water, as would be the case with many stream beds.

Having disposed of these details, a consideration of the question of uniqueness may be resumed. Professor Hrennikoff is correct when he states that the solutions are not unique. The writers would like to urge, however, that the difficulty is not due to the form of their proposed solutions but is inherent in the nature of the problem. This may be illustrated by the retaining wall problem. Here the active and passive states represent limiting states beyond which stress distributions of the linear type cannot exist. Between these limits, however, there are a multitude of elastic states possible, and it should be noted that all of them, including the active and passive cases, are solutions of the differential equations (Eqs. 3 and 12) which meet the boundary conditions. This does not include all the possibilities, since additional solutions of non-linear type probably can exist also. It is evident, then, that Rankine's active state solution for the case of no surcharge, for example, is not unique in the sense that this distribution must exist to the exclusion of all others. It remains a physical possibility, however, in spite of the fact that tests and experience show that greater horizontal pressures can exist. In practice, other factors besides the satisfaction of the differential equation and the boundary conditions must be taken into account when a decision must be made as to the design pressure of a retaining wall. These factors will include a consideration of the manner of placing the backfill and the conditions of service after completion.

It is necessary, also, to exercise an intelligent choice when selecting the stress states for the plastic zones in a dam. An active state for the downstream slope might be reasonable for a dam that is to be constructed by rolled-fill methods, whereas a hydraulic fill dam might require a different choice. These remarks will apply to the comments in Professor Hrennikoff's sections 1 and 4. The questions raised in section 3, regarding the use of Hooke's law, have already been dealt with in connection with Professor Hennes' discussion.

In regard to the question raised by Professor Hrennikoff in section 5, concerning continuity of stress, it may be stated that the choice here was a matter

of judgment. In the case of a dam subjected to a changing load, where the boundaries between elastic and plastic zones must move, it was considered that a discontinuity of the normal stress parallel to the boundary would lead to difficulties in regard to compatibility of strains on the elastic side of the boundary in the case where this zone is widening. In other cases, a choice of equality for shear stresses along the boundary and for direct stresses normal to it seems to be indicated. In Fig. 5, if the percolation pressure gradients in zone VII are raised a certain amount, the elastic zone VI will collapse to a simple boundary, subjected to such a stress condition. In Example 2, the percolation pressure portion is incomplete, as was stated in the original description. This is due to the limitations imposed by the available solutions. In this case, the choice of a percolation pressure was made so that the actual body forces would be represented as well as possible. To determine how seriously the validity of the solutions would be affected by these limitations, some tests were conducted on model dams. These were built in a special tank which was about 20 ft long, 4 ft deep, and 3 ft wide. The model dams were approximately 2 ft high and rested on a foundation about 2 ft deep. The case shown in Fig. 6(*c*) was tested on several occasions with a dam whose upstream and downstream slopes were 1 on 3 and 1 on 2, respectively. The computations indicated that a sliding failure should occur at the downstream toe, with an upward percolation pressure gradient in zone VII, which would be slightly less than that required to cause piping. The exact figures for one of the models tested were 113.25 and 123.10 lb per sq ft per ft to cause sliding and piping, respectively. The sliding failure was observed to take place at the expected gradient and before any material had been removed by piping. The estimated gradient to cause piping was later confirmed very closely. It was concluded from this test that the neglected portions of the percolation pressure pattern did not influence the results of the computations greatly.

It is only necessary to test one point inside an elastic region if the equality is satisfied at the two boundaries. The reason for this may be stated as follows: The equation for R_0 is second degree in x for any constant value of y (say y_1) when linear stresses and a uniform coefficient of friction are assumed. A plot of this equation will show intercepts on the x-axis at both $x_1 = \beta_i\, y_1$ and $x_2 = \alpha_i\, y_1$ when plastic regions bound the elastic zone along $x = \beta_i\, y$ and $x = \alpha_i\, y$. Therefore, to check this inequality, the condition $R > 0$ at only one point along the line between (x_1,y_1) and (x_2,y_1) is sufficient. The same reasoning would apply for any other constant value of y, such as y_2, y_3, etc.; therefore, to check the inequality at only one point within the entire region is sufficient.

Tables 1 and 2 of Appendix II were worked out by restricting solutions of Eq. 18 to real values so that stresses as given by Eqs. 21 would be real. The discriminant was set equal to zero, and values of n were determined for the various weights, coefficients of friction, etc., shown in the tables. The two systems of percolation pressure contours used are shown in Figs. 6(*a*) and 6(*b*). Table 3 was computed by determining the percolation pressure gradient in zone VII which would cause this zone to assume the passive state. For some-

what lesser gradients, zone VI is absent and zone VII is in the elastic state, as explained previously.

Professor Krynine presents a summary of present methods of analysis of earth structures, which he follows with a brief geometrical interpretation of conditions in the plastic regions. This latter interpretation, together with his explanations of some of the mathematics, should prove helpful to any one going through the method for the first time. His section showing that differential equations are not necessary in plastic regions is an interesting presentation but would not shorten the computations for such regions. It was postulated that the material in the elastic regions obeys Hooke's law, but this assumption need not be made if a more reasonable behavior can be stated. Whether Hooke's law is postulated, or rectilinear stress in the elastic region is assumed, makes no difference, of course, when the designer is limiting himself to linear solutions, but as stated in the conclusion to the paper a general method of approach was being developed. The procedure as used in the paper would hold even if neither Hooke's law nor rectilinear stress were assumed.

In closing, the writers wish to take this opportunity to express their sincere thanks to those who have contributed to the discussions.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

LATERAL STABILITY OF UNSYMMETRICAL I-BEAMS AND TRUSSES IN BENDING

Discussion

BY GEORGE WINTER, ESQ.

GEORGE WINTER,[12] ESQ.[12a]—In connection with the general tendency toward higher working stresses and more slender members in engineering structures, problems of the stability of such members are becoming increasingly important and are given wide attention in engineering research. Since the first publication of the writer's paper in December, 1941, three articles on related topics have been published. Prof. J. N. Goodier[13,14] has given a comprehensive and, in many respects, fundamentally new treatment of stability problems involving the simultaneous occurrence of compression, bending, and twist. Mr. Hill[15] has investigated the buckling of unsymmetrical I-beams in pure bending—that is, the same problem which forms part of the writer's paper. Whereas the writer derived his equations by means of energy principles, Mr. Hill used differential equations for this purpose. His discussion of the writer's paper represents an abstract of his aforementioned article.[15]

For the case of pure bending, Mr. Hill derives Eq. 71 which, in form, differs from Eq. 14*a* for the same case. As Mr. Hill rightly points out, his equations are rigorously exact only if the centroid of the section is midway between the flanges. The same holds true for the writer's equation. This does not mean that these equations apply to symmetrical beams only. Although the areas of the flanges must be equal in this case, the $\frac{b}{t}$-ratios for the two flanges may be different (b = width, t = thickness). In many instances

NOTE.—This paper by George Winter, Esq., was published in December, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1942, by Robert K. Schrader, Jun. Am. Soc. C. E.; and March, 1942, by H. N. Hill, Assoc. M. Am. Soc. C. E.

[12] Asst. Prof. of Civ. Eng., Cornell Univ., Ithaca, N. Y.

[12a] Received by the Secretary September 14, 1942.

[13] "The Buckling of Compressed Bars by Torsion and Flexure," by J. N. Goodier, *Bulletin No. 27*, Cornell Univ. Eng. Experiment Station, Ithaca, N. Y., December, 1941.

[14] "Flexural-Torsional Buckling of Bars of Open Section," by J. N. Goodier, *Bulletin No. 28*, *ibid.*, January, 1942.

[15] "The Lateral Instability of Unsymmetrical I-Beams," by H. N. Hill, *Journal of Aeronautical Sciences*, Vol. 9, No. 5, March, 1942.

beams are so designed in structural practice, especially to avoid local crippling of the compression flange due to large $\frac{b}{t}$ -ratios.

The writer is gratified to find that, for the case of equal flange areas (the only one for which both equations are strictly exact), Mr. Hill's and his own expressions are identical. To demonstrate this fact let Fig. 9 represent an I-beam with a centroid C, a flexural center F, and a midpoint M. Then the distance e, introduced by Mr. Hill, is the distance from C to F. For equal flange areas the points C and M coincide. In this case, with the notation of the paper and Mr. Hill's rule of signs,

$$e = -\left(\frac{h}{2} - c\right) = \frac{h}{2}\,\frac{I_1 - I_2}{I} \quad \ldots\ldots (72)$$

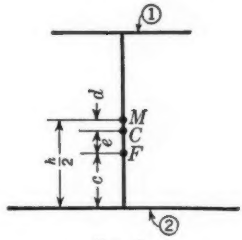


FIG. 9

Substitution of this expression in the second term of Eq. 71 gives:

$$I\,\frac{2\,e}{h} = I_1 - I_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (73)$$

which is identical with the second term of Eq. 14a. Likewise, substituting the value of e in the fourth term of Eq. 71, and combining it with the third term, results in:

$$4\,I_1\,I_2 + I^2\,\frac{4\,e^2}{h^2} = I^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (74)$$

which, in turn, is identical with the corresponding term in Eq. 14a. For this case, then, Mr. Hill's and the writer's expressions are the same.

If C and M do not coincide, neither Mr. Hill's nor the writer's equations are strictly exact. In this case the difference between the two approaches is that Mr. Hill implicitly takes account of the distance d from the centroid C to the midpoint M. However, he neglects the quantity $\frac{Z_Y}{I_Y}$ (see Eq. 67). The magnitude of d is found from

$$d = \frac{h}{2}\,\frac{A_1 - A_2}{A_1 + A_2 + A_w} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (75)$$

in which A_1, A_2, and A_w are, respectively, the areas of the two flanges and of the web. It is easily seen that the quantity $\frac{Z_Y}{I_Y}$ is of the same order of magnitude as d. Therefore, little is gained in accuracy by taking account of d but neglecting $\frac{Z_Y}{I_Y}$. Except for the rare case of radically different values of A_1 and A_2, the quantity d is seen to be rather small as compared with $\frac{h}{2}$ or c. Therefore, the writer's simple Eq. 14a represents a close approximation even for the case in which the flange areas are different.

The practical advantage of the writer's work-energy method is that it provides a rather simple way of analyzing the stability of beams under transverse loading rather than only in pure bending. The writer's equations for pure bending and for center load are supplemented in Lieutenant Schrader's contribution by those for uniformly distributed loading and for two equal loads symmetrically located on the beam, derived by the writer's method. An investigation of these cases by means of differential equations would have met with practically insurmountable mathematical complexities. It is believed that the four types of loading analyzed cover the most important practical cases.

In closing the author wishes to draw attention to the fact that the results of his and other related investigations are directly applicable only if the maximum stress due to the critical load is smaller than the proportional limit of the material. This limitation, the same as that of the Euler formula for columns, is common to all calculations concerning the elastic stability of structures.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

TIMBER FRICTION PILE FOUNDATIONS

Discussion

BY FRANK M. MASTERS, M. AM. SOC. C. E.

FRANK M. MASTERS,[21] M. AM. SOC. C. E.[21a]—It was the author's idea to present, in this paper, test data which seemed to demonstrate some facts about group action in timber friction piles that previously had been largely a matter of conjecture. The tentative theory for the calculation of failure loads of groups proposed in the paper appeared to check the test results and therefore seemed to have some merit. Such a theory can be of value only if it meets all conditions and if it is improved by criticism. The writer, therefore, sincerely appreciates the resulting comment and criticism.

Mr. Connor offers a number of practical suggestions on pile spacing that are too often neglected by the designer. He also gives data on heavy loads actually carried by friction piles driven into soft material which, during driving, did not appear to be able to support such loads. Mr. Connor further presents an interesting method of comparing the strengths of different groups of piles on the basis of comparative perimeter areas of soil around the groups. This method evidently gives a close approximation as between groups; but it is not possible to use this method to compare single piles with groups.

Mr. Woodruff raises several points of interest. The writer agrees that all predictions of pile strength using this theory will not have the accuracy of agreement between theory and tests apparently indicated in Table 5; nor does it seem that such accuracy is altogether necessary in the design of pile foundations. Further, it is quite possible, as Mr. Woodruff suggests, that soil consolidation around the piles over a long period of time may affect their settlement somewhat; but it does not seem likely that such a long-time consolidation should affect the shear strength of the soil adversely and lead to pile failure. It is anticipated that settlement readings will be made on the Morganza Floodway structures over a period of time. It is to be noted in connection with

NOTE.—This paper by Frank M. Masters, M. Am. Soc. C. E., was published in November, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1942, by E. H. Connor, M. Am. Soc. C. E.; April, 1942, by Messrs. Glenn B. Woodruff, Jacob Feld, G. G. Greulich, and G. S. Paxson; May, 1942, by Raymond D. Mindlin, Assoc. M. Am. Soc. C. E.; June, 1942, by John W. Solomon, Esq.; and September, 1942, by A. E. Cummings, M. Am. Soc. C. E.

[21] Cons. Engr. (Modjeski & Masters), Harrisburg, Pa.

[21a] Received by the Secretary September 17, 1942.

long-time loadings that one of the pile groups on the N. O. T. & M. Crossing, group T21–T29, was loaded over a period of 243 days with a constant load of 20.3 tons per pile without apparent additional settlement after the first day.

Mr. Feld suggests that a tapered timber pile actually may carry with it, in failing, a complete cylinder of soil having a diameter equal to the butt diameter of the timber pile. He points out that under this assumption the shearing value of the soil at failure would be 600 lb per sq ft and would be more nearly in correspondence with the value of 558 lb per sq ft indicated in Table 4 for parallel-sided concrete piles. He also describes an approximate rule for determining group strengths by subtracting progressive one sixteenths from the value of all piles after the first, as every new pile is added to the group. This apparently gives satisfactory results for the examples cited, but does not take account of pile spacing in relation to length or diameter, which the writer believes has an important bearing on the strength of piles in a group.

Mr. Greulich states that the law of averages probably contributed to the agreement between theory and tests as recorded in Table 5, and with this the writer agrees. Mr. Greulich cites an interesting example of group action in piles under the large foundations of pivot piers of swing bridges, where an earlier designer found he must restrict working loads to as low as $7\frac{1}{2}$ to 8 tons per pile.

Mr. Paxson gives a graphic comparison of pressures beneath pile foundations as compared with pressures beneath spread foundations, showing the considerably greater pressure under the spread foundation, and giving emphasis as well to the fact that this greater pressure is at a point near the surface where soil conditions are generally poorer than near the elevation of pile tips.

Professor Mindlin states that a solution of the equations of elasticity, known as the Boussinesq solution, was used in the writer's paper, and that the integrated Boussinesq formula is not applicable to pile foundations. It is not to be inferred that, because the writer used the Boussinesq solution, he accepts the concept of elasticity in soils. The Boussinesq solution was used because it appeared to fit satisfactorily the results of tests to failure better than did any other formula. Nowhere in the writer's paper was there any mention of the word "elasticity"; this was a deliberate omission because it is not believed that soils are perfectly elastic. This is especially true in connection with the tests given in the paper which were carried to failure, well beyond any possible range of elasticity in the soil.

Professor Mindlin states that it is not possible to compute a curve of Mindlin distribution for a uniform load per linear foot from the data given in Table 10. The method used in the preparation of Fig. 11 was as follows: Part (*a*) of Table 10 gives the pressures at various radii from the pile for various depths. These data permit a computation of the total vertical force on a circle of 0.1 radius, 0.2 radius, 0.3 radius, 0.5 radius, etc., at such depths. For a pile transmitting a uniform load per linear foot to the soil, the total load distributed from the top to 0.25 L depth is $\frac{1}{4}$ P, and to 0.50 L depth is $\frac{1}{2}$ P. If, from these distributed loads, there are subtracted the total vertical forces acting on the circles of various radii already cited, the difference in each case is the total shear carried by the cylindrical surface of the cylinder of earth from the top of pile to

the depth 0.25 L or 0.50 L. For each radius a curve of total shear may be drawn, starting from 0 at the top to a calculated value at 0.25 L, to a larger calculated value at 0.50 L. The slope of this curve at any point is the shear transmitted by a circumferential band one unit high. If this slope is divided by $2\pi r$, it will give the shearing intensity per square unit. The curve of Mindlin distribution in Fig. 11 was based on the slope of the various curves at midheight of the pile. Any error that exists as a result of the graphical method used is believed to be slight.

Professor Mindlin questions the principle of superposition applied to pile groups. Of course, there are many unusual stresses created between piles in a group due both to the existence of the other piles and to soil stresses created during driving. Nevertheless, there seems to be no reasonable way of determining the build-up of stresses under pile loads unless loads from various piles are superimposed upon one another.

Professor Mindlin states that the real problem is to determine the distribution along the pile of the load transfer from pile to soil. There are certain practical ways of arriving at this solution. When a timber pile is started in the ground and the hammer is placed upon it, it will often settle into the soil a distance of several feet. In the case of the Morganza piles, this distance of embedment into the soil under the weight of the hammer used was such as to suggest that the shearing strength of the soil near the surface was reasonably close to the average shearing strength at failure of the pile. As was emphasized in the paper, when some allowance is made for the comparatively larger diameter of the butt as compared with the tip of a pile, and when there is reason for the belief that the shearing strength of soil increases slightly with depth, it is reasonable to arrive at such a value of R_c as was used in the paper. Incidentally, such practical data as these, secured during driving operations, may often give a clue as to the variations in material from ground to tip of pile. In a recent study undertaken in the writer's office, dealing with a pile foundation at another locality, it was discovered that this sinking of the pile under the weight of a 9,700-lb hammer amounted to 15 ft, indicating a shearing strength of soil at the surface of the ground of only 400 lb per sq ft, whereas the entire pile failed under an average skin friction of 975 lb per sq ft. This indicated a considerable variation in strength of material and suggested a method of arriving at a value of R_c for the pile group in question.

Mr. Solomon demonstrates mathematically the effect of the elasticity of the pile upon R_c and proves that for small pile loads the pile tip actually may deliver no load to the soil. Since the shearing stresses computed in the paper were primarily related to failure loads on the piles, the value of R_c used must be considered as applying particularly to such failure loads. Mr. Solomon also suggests that some of the difference in skin friction value between concrete and timber piles may be due to the effect of weight of the concrete piles as compared with the weight of timber piles. All of the larger concrete piles were hollow and had a weight no greater than the weight of the earth which they displaced. If a cylinder of earth is visualized as standing in soil, there will be no vertical shear along its vertical surfaces except that created by an applied load on top

of the cylinder. For that reason, the weights of the piles themselves were uniformly and properly neglected in computing skin frictions under loads.

Mr. Cummings declares that the Boussinesq solution is not applicable to piles in an elastic soil, and shows that a combination of piles and soil is not isotropic. He restates the effect of contraction of the pile upon the distribution of load to the soil, and questions the equality of loads on the various piles of a group in the Morganza tests. Mr. Cummings evidently feels that not enough measurements were made and that the Morganza tests were not up to the experimental standard established by Galileo. In that connection, although the writer has heard many suppositions as to the effect of grouping upon pile strength, he knows of no previous tests of the scope of the Morganza tests to establish by measurement what this grouping effect is at failure. The writer also wishes to restate a fact which appeared in the paper—that these tests were originally intended to satisfy the requirements of current specifications, and that certain failures of small groups suggested that the tests be extended to larger groups. The writer is indebted to the Corps of Engineers, U. S. Army, for their cooperation in making possible these supplementary failure tests on large groups. The theory was then developed, using that method of distribution which would check the tests to failure and, by such a practical check, the method served to cancel the numerous variables which are cited by Mr. Cummings.

Mr. Cummings also objects to the use of average value of shear strength over the several crossings in cases where single pile tests were not close at hand. This was done because it seemed the most reasonable approach, since funds had not been available to make a single pile test close to each group test. However, after the paper was published, the N. O. T. & M. Railway, at its own expense, made a failure test on a single 50-ft pile immediately adjacent to the 16-pile group, T37–T52, and obtained a failure shear value of 735 lb per sq ft at this location. The 16-pile group used 60-ft piles, which might be expected, due to their increased depth, to develop somewhere between this value of 735 and the average value of 778 lb per sq ft used in the study.

In conclusion, the writer hopes that opportunity may arise to study the effect of grouping by all of the methods proposed, both in the discussions and in the paper, and urges that every effort should be made to relate all theories developed to actual load tests in the field.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

DESIGN OF ST. GEORGES TIED ARCH SPAN

Discussion

BY C. H. GRONQUIST, ASSOC. M. AM. SOC. C. E.

C. H. GRONQUIST,[13] ASSOC. M. AM. SOC. C. E.[13a]—Preliminary design of the St. Georges tied arch span was made by assuming the 3-ft arch rib as entirely flexible, leaving the 9-ft tie girder to resist the live-load bending moments without assistance from the rib. Under this assumption the action of the structure is exactly analogous to that of the self-anchored suspension bridge, inverted. The flexible arch rib, acting only in direct compression, corresponds to the cable of the self-anchored suspension bridge, whereas the tie girder, acting both in direct tension and in bending, corresponds to the stiffening girder of the self-anchored suspension bridge, which must also transmit the horizontal component of cable stress as a thrust over the entire length of the bridge.

The action of the two structures being analogous, the equations and method of analysis of the self-anchored suspension bridge can be applied with only slight modification to the analysis of the tied arch with a flexible rib. One point of difference in the action of the two structures is to be found in the effect of vertical curve of roadway and tie girder or stiffening girder. The vertical curve of the stiffening girder will decrease stresses in the self-anchored suspension bridge, but the stresses in the tied arch are increased by the vertical curve. Since both structures are of the "closed" type, resembling simple trusses to that extent, the change in dimensions produced by deflection has no important effect on the stresses determined by the elastic theory using the dead-load dimensions, in the case of either the self-anchored suspension bridge or the tied arch. This is in distinction to the effect of deflection on both the externally anchored suspension bridge and the arch bridge.

A more complex but similar structure consisting of a 3-span continuous girder with the center span reinforced by a flexible arch rib tied to the girder is analogous to the self-anchored suspension bridge with unloaded side spans,

NOTE.—This paper by J. M. Garrelts, Assoc. M. Am. Soc. C. E., was published in December, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1942, by R. W. Abbett, M. Am. Soc. C. E.; April, 1942, by Jacob Karol, Esq.; May, 1942, by Alexander Dodge, Esq.; June, 1942, by Carlos A. Bejarano, Jun. Am. Soc. C. E.; and September, 1942, by Messrs. A. A. Eremin, and A. M. Freudenthal.

[13] Associate Engr., Robinson & Steinman, Cons. Engrs., New York, N. Y.

[13a] Received by the Secretary September 1, 1942.

except that in the case of the tied arch bridge the side-span girders will not act in direct stress. Again, however, the analysis of the two structures could be made by application of the same methods and equations.

Professor Garrelts gives no derivation for Eq. 17 for the final analysis of the St. Georges tied arch span. Although this may have been assumed to be obvious, the value of this excellent paper may perhaps be increased by clarifying somewhat the theoretical basis for the final analysis. Hanger elongation under live load was considered negligible, as were the tangential components of deflection. Consequently, the angle between the rib and girder at the end of the rib, as well as elsewhere, is not changed during deflection of the structure, and, therefore, there can be no appreciable moment in the rib at its end connections with the girder. The span can be made determinate by cutting the rib at the girder connections so as to eliminate H, the horizontal component of direct stress in the rib and girder. The structure then will consist of two simple beams, the arched rib, and the girder, which are connected by the inextensible hangers and which must participate in proportion to their effective moments of inertia in carrying the live load.

With only the value of the horizontal component of direct stress to be determined, the structure is singly indeterminate, as for the preliminary design. The moment in the structure for a unit value of this redundant is $m = y - y'$, this moment also being divided between the rib and the girder in proportion to their effective moments of inertia. The direct stress in the rib for a unit value of H is $u = \sec \alpha$, assuming the hanger loads to be equal throughout, and the direct stress in the girder is of course $u = 1$. Eq. 17 for H then follows from the substitution of the foregoing moments and direct stresses in the conventional Eq. 2 for a single redundant.

Final analysis of the previously mentioned tied-arch continuous girder can be made by this same method. By taking the basic determinate system as the combination of the continuous girder and the center span simple arched beam, there would remain only the value of H, the horizontal component of direct stress, to be determined by Eq. 17. The actual values of M' in this equation, however, could be taken for the simple span condition, and therefore only for the center span. Similarly, the first term in the denominator could be written $\frac{\Sigma\, m\, m_0\, \Delta l}{I_{tr}}$ in which m represents the moment due to a unit value of H in the basic continuous structure, and m_0 represents the moment due to $H = 1$ in the center span as a simple structure. This follows from the fact that in computing the deflection of an indeterminate structure by the method of the dummy unit load, the value of either M', the moment for the basic determinate condition, or m, the moment produced by the unit load, may be taken for a statically determinate condition. In applying Eq. 18 to obtain the final moments, M', of course, would be taken for the basic continuous system.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

PROFILE CURVES FOR OPEN-CHANNEL FLOW

Discussion

By Messrs. J. C. Stevens, and Boris A. Bakhmeteff and Nicholas V. Feodoroff

J. C. Stevens,[10] M. Am. Soc. C. E.[10a]—When the writer began reading Professor Gunder's paper, he thought: "Here's just what we've been looking for"; and, when he had read as far as his general differential equation for the surface profile (Eq. 13), it appeared so innocent that he thought: "We'd better turn real mathematicians loose on some of our other problems." Then, on turning the page, that innocent looking thing the Professor states "is not difficult to integrate" loomed out of the darkness like the iceberg did to the *Titanic* and wrecked everything. It is suggested that the Professor be sentenced to prepare a table of values of his K-factor, in terms of y, y_c, and y_o, varying by tenths of feet to 50 ft. One wonders how large a volume it would make.

However, it does prove that, defining the friction factor by the Manning relation (which is purely empirical and dimensionally incorrect), the surface profiles, determined for a unit width of a wide channel (which does not exist), contain points of inflection in the range where depths are less than either normal or critical, in which range the Manning formula does not apply! Professor Gunder did quite well with his disclaimers, but he is challenged to beat that one.

After all, it appears as though hydraulic engineers will just have to struggle along with their old step-by-step method of computing water-surface profiles.

Boris A. Bakhmeteff,[11] M. Am. Soc. C. E., and Nicholas V. Feodoroff,[12] Esq.[12a]—A serious error that has remained unnoticed in hydraulic literature for years has been disclosed by Professor Gunder. As correctly surmised by the author, the shape of the so-called M_3 surface curve, traditionally presented in textbooks, dates back to the past century, when varied flow profiles were

Note.—This paper by Dwight F. Gunder, Esq., was published in April, 1942, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1942, by Carl Rohwer, Assoc. M. Am. Soc. C. E.; and September, 1942, by C. J. Posey, Assoc. M. Am. Soc. C. E.

[10] Cons. Hydr. Engr. (Stevens & Koon), Portland, Ore.

[10a] Received by the Secretary July 13, 1942.

[11] Prof., Civ. Eng., Columbia Univ., New York, N. Y.

[12] Research Associate, Civ. Eng. Dept., Columbia Univ., New York, N. Y.

[12a] Received by the Secretary August 19, 1942.

first classified by considering the special case of broad rectilinear channels, with the further simplifying premise of a constant average Chézy factor C applied over the entire range of depths. Under these special conditions, the outline of the M_3-curve remains concave $\left(\frac{dy}{dx} > 0; \frac{d^2y}{dx^2} < 0\right)$ for all the depths, tending toward $\frac{dy}{dx} = \infty$ as y approaches the critical value. A further circumstance to be taken into account is that within the customary reaches the M_3-profiles happen to be very "flat," exhibiting (except near y_c) only slight curvature and, on the whole, approximating a straight line. This means that the changes in curvature, as disclosed by Professor Gunder, were not conspicuous and the observed appearances did not upset accepted notions. Therefore, the over-all concave M_3-profile, initially deducted for specific conditions, was inadvertently extended and assumed to apply in all cases, without due investigation of the special effects devolving from a variable friction factor. For such unpardonable negligence, the senior writer, as the author of a manual especially devoted to varied flow, presents his humble apologies. It should be stated, nevertheless, that at least in some texts[13] the tracings of the M_3-curve in the conventional surface profile diagrams are "stopped" at a certain distance from the bottom, indicating that in the vicinity of $y = 0$ the deductions drawn from the general varied flow equation cannot apply.

Professor Gunder refers to the features disclosed by him as "irregularities." It is only proper to qualify previous ideas as plainly erroneous, and to accept the author's discoveries as regular characteristics of the M_3 varied flow curves, especially as these properties seem to have a broader significance than that suggested by the paper, and as they will be found to obtain quite generally, irrespective of the particular formula used for the C-factor.

The following analysis complements Professor Gunder's pioneer findings, which from now on should replace all former ideas relating to M_3-curves. Also, his discoveries are interpreted in the light of experimental evidence.

(1) The first deduction, drawn by Professor Gunder from Eq. 13, is that the surface curve for broad rectilinear channels with Manning coefficients theoretically becomes vertical at $y = 0$. As a matter of fact, this property is not to be limited to any particular form of channel or to any specific friction formula. Indeed, it constitutes a general characteristic which applies in all instances. In the general varied flow equation[14]

$$\frac{dy}{dx} = S_o \frac{1 - \dfrac{A^2_o\, R_o\, C^2_o}{A^2\, R\, C^2}}{1 - \dfrac{Q^2\, b}{g\, A^3}} \qquad (23)$$

when the depth approaches zero, the unity factors both in the numerator and denominator become small by comparison to the second members, and can be omitted. Accordingly, with $S_o\, A_o\, R_o\, C_o = Q$, the limit of the surface

[13] "Hydraulics of Open Channels," by Boris A. Bakhmeteff, McGraw-Hill Book Co., Inc., New York, N. Y., 1932, p. 76.

[14] *Ibid.*, Eq. 21, p. 31.

gradient

$$\text{Limit}\left(\frac{dy}{dx}\right)_{y\to 0} = \frac{\dfrac{Q^2}{A^2\,R\,C^2}}{\dfrac{Q^2\,b}{g\,A^3}} = \frac{g}{C^2}\,\frac{p}{b} = \sigma \ldots\ldots\ldots\ldots\ldots (24)$$

in which σ is the critical slope.[15] For small depths, $\frac{p}{b}$ tends toward unity. Hence,

$$\text{Limit}\left(\frac{dy}{dx}\right)_{y\to 0} = \frac{g}{C^2} = \frac{c_f}{2} \ldots\ldots\ldots\ldots\ldots\ldots (25)$$

in which c_f is the dimensionless friction drag coefficient in the average wall stress expression

$$\tau_o = \rho\, c_f\,\frac{V^2}{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (26)$$

By reason of its physical essence, as well as theoretically in the customary Bazin, Ganguillet-Kutter, Manning, etc., relations, the C-factor for $y \to 0$ tends to become zero, making the critical slope $\left(\text{and thus } \frac{dy}{dx} \text{ in Eq. 25}\right)$ infinite. Note that this feature does not depend on the value of the channel slope S_o. In other words, all surface curves, both of the M_3 and the S_3 class, tend to be theoretically vertical at $y = 0$.

(2) As Professor Gunder infers, properties derived from the $y \to 0$ region represent purely theoretical conditions. The important practical effect, nevertheless, is that in the reach of small depths the M_3-curve must be convex $\left(\frac{d^2y}{dx^2} < 0\right)$, and that therefore at some depth y_i, between $y = 0$ and $y = y_c$, there is an inflection of curvature, with the surface profile changing from convex to concave. The paper conveys the impression that the eventual location of y_i is particularly affected by friction coefficients expressed through the Manning formula. In fact, the paper suggests (line 12 following Eq. 14) that "* * * in the range of type 3 curves, the Chézy coefficient C should be determined either by the Bazin or Ganguillet-Kutter relation and not by the Manning relation." The recommendation apparently is actuated by the desire to avoid the noted "irregularities."

One should not try to escape facts. What really matters is whether the property in question is actual or not; and under all circumstances the selected formula should be the one that is known to comply best with established reality. Unfortunately, very little is known about resistances in rapid (shooting) motion. In such a region investigators use formulas and coefficients deduced from observations in tranquil motion ($y > y_c$) without any direct assurance that they apply equally to supercritical ($V > V_c$) flow. Moreover, for "slowly varied (parallel) motion,"[16] they assume that the friction rate

[15] "Hydraulics of Open Channels," by Boris A. Bakhmeteff, McGraw-Hill Book Co., Inc., New York, N. Y., 1932, p. 48.

[16] *Ibid.*, pp. 8, 24, and 29.

for a certain discharge and depth is substantially equal to what the resistance would have been under uniform conditions. In the M_3-case, flow is divergent and the divergence at times becomes quite pronounced. Analogous to closed conduits, one may anticipate a complementary loss connected with the restoration of kinetic energy into potential energy. Again, there is no material available at present that would evidence the magnitude of such restoration losses in open-channel flow.

Accordingly, in many respects the precise answers to the questions raised by the paper will have to await evidence from future research. In the meantime, one is forced to seek approximate solutions by recourse to one or another of the existing friction formulas, and it is here that the writers cannot quite follow Professor Gunder's conclusions, for their investigations seem to show that the choice of any relation for C does not materially affect the position of the inflection point.

(3) The location of the "inflection depth" y_i on the M_3-curve, where the curvature changes from convex to concave, can be expressed conveniently in dimensionless terms by the ratio

$$\frac{y_i}{y_c} = \frac{\text{inflection depth}}{\text{critical depth}} = (y'_c)_i \qquad (27)$$

Generally, the y_i-value will be influenced by the shape of the channel, the discharge, the slope S_o, and the wall roughness. For two-dimensional motion (broad, rectangular channel), the shaping agencies are reduced to the parametric ratio

$$\frac{y_c}{y_o} = \frac{\text{critical depth}}{\text{normal depth}} = \eta_c \qquad (28)$$

and to the form of the function

$$C = f(y) \qquad (29)$$

which depicts the change of the Chézy factor C with the varying depth of flow.

To apprehend the possible effect of the different formulas for C, it is necessary to make clear the physics that actually underlie the structure of all empirical relations. In all formulas, a "surface roughness type" is characterized by a proper numerical term, such as the n-factor in the Ganguillet-Kutter and the Manning expressions. Such a friction term is held to be invariant for all conduits involving a particular type of surface roughness. Each formula offers the friction term with a mathematical expression deemed to represent best the factual change of C with the conduit size. In all instances C increases with the latter, the implied physical meaning being that the friction term n exemplifies a certain roughness size, and that the increase of C with the hydraulic radius R reflects the eventual reduction of the friction drag with the decrease of the relative roughness. In fact, relative roughness appears in the empirical relations in the form of a ratio of the friction term to an appropriate power of the hydraulic radius. Thus, the Bazin and Kutter expressions are built around the ratio of the friction term to $\sqrt{R}$, whereas in the Manning formula the shaping ratio is $\frac{n}{R^{1/6}}$. In general, the aim of "empirical" research

in the past was to evolve a mathematical form between C and R, which for a certain "n" would best average and fit the maze of observational data assumed to respond to the same roughness type. In essence, the purpose was essentially the same, as answered more recently by the Prandtl-Kármán semi-rational theory, which expressed the interrelation between $\frac{1}{\sqrt{c_f}} = \frac{C}{\sqrt{2g}}$ and the relative roughness $\frac{e}{R}$ in logarithmic terms.

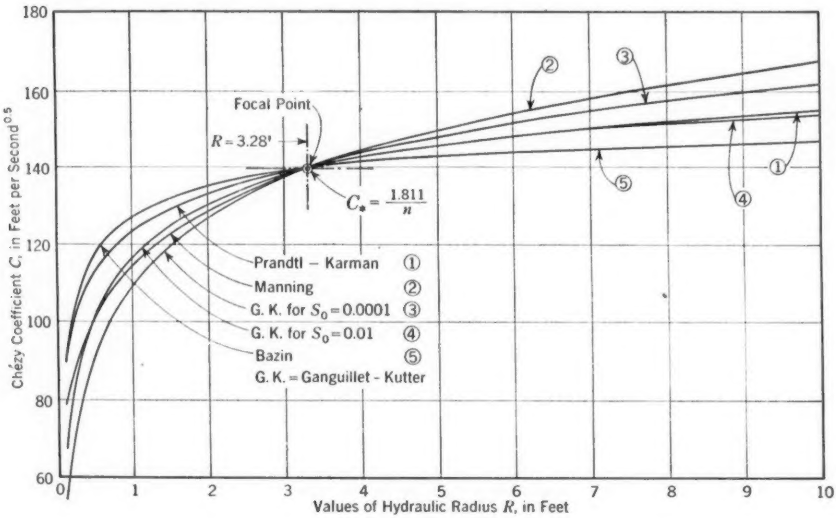


FIG. 3

A comparison of the different suggestions is given in Fig. 3, which shows a group of $C = f(R)$-curves computed by the different formulas with the Ganguillet-Kutter term $n = 0.013$. It should be remembered that, in the otherwise rather complex and obviously artificial Ganguillet-Kutter structure, the value of C_*, for $R = 3.28$ ft = 1 m, constitutes a focal point at which the Chézy factor is assumed to be unaffected by the slope S_o, and in metric measures is numerically equal to $C_* = \frac{1}{n}$. For feet, the focal value is $C_* = \frac{1}{n}\sqrt{3.28} = \frac{1.811}{n}$. It is the same focal point that conditions the Manning curve, the metric form for which is

$$C = C_* R^{1/6} = \left(\frac{1}{n}\right) R^{1/6} \qquad (30a)$$

which is expressed in feet as

$$C = \left(\frac{1.811}{n}\right)\left(\frac{R}{3.28}\right)^{1/6} = \left(\frac{1.49}{n}\right) R^{1/6} \qquad (30b)$$

On the other hand, the logarithmic expression

$$C = (C_* - 16.5) + 32.1 \log R = \frac{1.811}{n} - 16.5 + 32.1 \log R \ldots . (31)$$

represents, in foot units, the Prandtl-Kármán curve, customarily referred to as the Nikuradze formula, adapted to pass through the Ganguillet-Kutter focal point for the roughness term n. (In the well-known expression for rough pipes,

$$1 \sqrt{f} = A_r + 2 \log \frac{r_o}{e} \ldots \ldots \ldots \ldots \ldots \ldots . (32)$$

replace $\frac{1}{\sqrt{f}} = \frac{1}{\sqrt{4 c_f}}$ by $\frac{C}{\sqrt{8 g}}$, and r_o by $2R$. This gives

$$C = \sqrt{8 g} \left(A_r + 2 \log \frac{2}{e} \right) + 2 \sqrt{8 g} \log R \ldots \ldots \ldots (33a)$$

or, in foot units,

$$C = A_c + 32.1 \log R \ldots \ldots \ldots \ldots \ldots \ldots . (33b)$$

For $R = 3.28$ ft, C must equal $C_* = \frac{1.811}{n}$. Hence, the constant $A_c = C_* - 32.1 \log 3.28 = \frac{1.811}{n} - 16.5$.) Finally, the Bazin curve $C = \frac{158}{1 + \frac{M}{\sqrt{R}}}$ is fitted to pass through the focal point by making the friction term $M = 158\,n - 1.811$.

In Fig. 4 the C-values are plotted logarithmically. As seen, the curvature of the outlines becomes rather unpronounced, and one is naturally tempted,

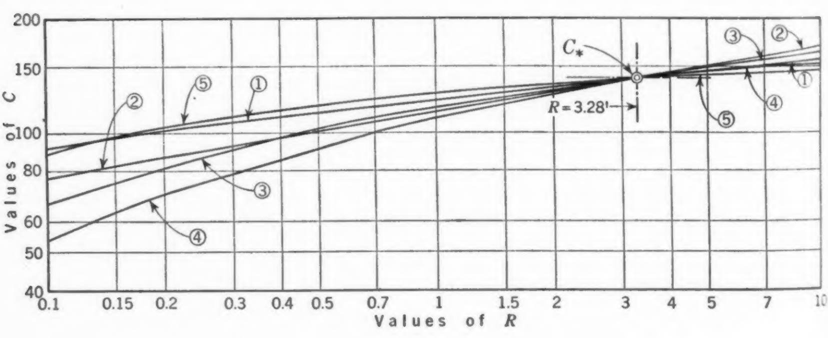


FIG. 4

for example, to replace the complex Ganguillet-Kutter relation by a suitable straight line answering the exponential relation $C = C_* R^p$, passing through the focal point C_*, and adjusted, so far as the incline p is concerned, to approximate best the $C = f(R)$-curve in the desired region. This explains the origin and the significance of the Manning formula, which is fitted to conform with $p = \frac{1}{6}$.

More generally, in the light of Fig. 4, one may state that the various relations of C versus R differ by the relative steepness of their incline. Therefore, if an exponential expression were to be used for approximating the $C = f(R)$-function in that or any other reach of the curve, the respective formulas could be said to differ by a larger or smaller value of the exponent. The particular effect on Eq. 23 would be the substitution, in the numerator, of the ratio $\left(\frac{R_o}{R}\right)^{2p}$ for $\left(\frac{C_o}{C}\right)^2$.

(4) In the particular instance of a broad rectangular channel (Eq. 11*a*), one obtains, with such substitution,

$$\frac{dy}{dx} = S_o \frac{1 - \left(\frac{y_o}{y}\right)^{3+2p}}{1 - \left(\frac{y_c}{y}\right)^3} = S_o \frac{1 - \left(\frac{y_o}{y}\right)^m}{1 - \left(\frac{y_c}{y_o}\right)^3 \left(\frac{y_o}{y}\right)^3}$$

$$= S_o \frac{1 - \left(\frac{y_o}{y_c}\right)^m \left(\frac{y_c}{y}\right)^m}{1 - \left(\frac{y_c}{y}\right)^3} \quad \text{.................(34)}$$

The possible influence of friction, as appraised by different formulas, depends on determining the value of $(y'_c)_i = \frac{y_i}{y_c}$, which in Eq. 34 would correspond to the different values of the "hydraulic exponent," $m = 3 + 2p$, subject to the parametric values of $\eta_c = \frac{y_c}{y_o}$. One could proceed directly by using the third quantity in Eq. 34; but the numerical computations prove to be easier if one first finds the respective $\eta_i = \frac{y_i}{y_o}$-values, and then determines the required $(y'_c)_i = \frac{y_i}{y_c}$-quantities from $(y'_c)_i = \frac{\eta_i}{\eta_c}$. In fact, differentiating the second quantity in Eq. 34, and making $\frac{d^2y}{dx^2} = 0$, one obtains, after reduction,

$$\eta_c = \frac{y_c}{y_o} = \sqrt[3]{\frac{m\,\eta^3_i}{3\,(\eta^m_i - 1) + m}} = \eta_i \sqrt[3]{\frac{m}{3\,\eta^m_i + (m-3)}} \quad \text{.....(35)}$$

The solution for a series of exponents $m = 3 + 2p$, ranging from $p = 0.1$ to $p = 0.4$, thus embracing all the possible slopes in Fig. 4, is presented in Fig. 5(*a*). Fig. 5(*b*) shows the final values of $(y'_c)_i = \frac{y_i}{y_c}$ versus $\eta_c = \frac{y_c}{y_o}$. In particular, the values of $(y'_c)_i$ for the vertical $\eta_c = 0$, which corresponds to flow over a horizontal bed, are found from Eq. 11*b*:

$$\frac{dy}{dx} = -\frac{g\,y^3_c}{C^2\,y^3\left(1 - \frac{y^3_c}{y^3}\right)} = -\frac{g\,y^3_c}{C^2_*\,y^m\left(1 - \frac{y^3_c}{y^3}\right)} \quad \text{.........(36)}$$

which, when differentiated and solved for $\frac{d^2y}{dx^2} = 0$, gives

$$(y'_c)_i = \left(\frac{y_i}{y_c}\right)_{S_o=0} = \sqrt[3]{\frac{m-3}{3}} \quad \text{.................(37)}$$

As seen, the $(y'_c)_i$ versus η_c outline is rather flat, except when η_c approaches unity—that is, when the slope S_o becomes close to the critical, and the normal depth tends to become critical. Therefore, under ordinary circumstances, for

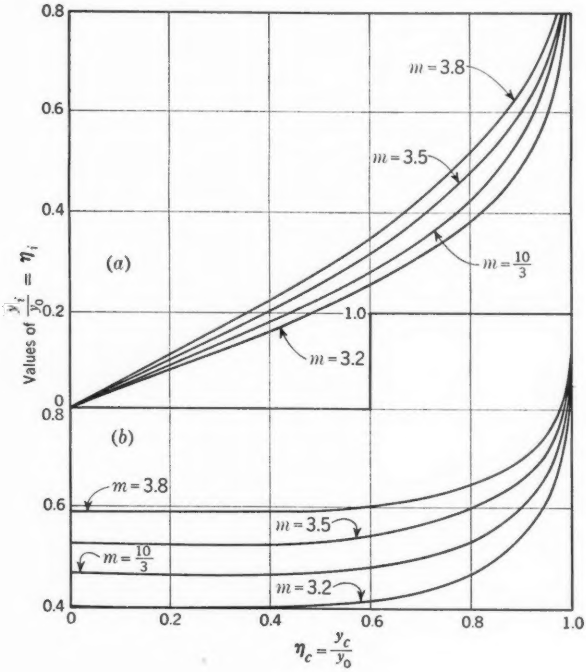


Fig. 5

$y_c \lessgtr 0.6\, y_o$, the simpler expression (Eq. 37) can be conveniently used for quick appraisals. On the other hand, for all possible values of the exponent, the inflection depth y_i remains within 0.4 to 0.6 of the critical, showing remarkable steadiness in its position. Thus, a rule of thumb, assuming inflection at about half the critical depth, seems to be a justifiable approximation.

(5) The purpose now is to disclose in a more direct fashion the exact possible effect of that or some other friction formula. Also, in view of the "flatness" of the M_3-curves, it is desirable to know more about their eventual outline in the regions adjoining the inflection point. The general procedure would be cumbersome and ineffective, so recourse must be had to practical examples. Calculations were performed for a wide range of circumstances, assuming two-dimensional flow at y_o = 10 ft, and making y_c successively 2 ft,

5 ft, and 8 ft. Using Eq. 11*a* in the form

$$\frac{\dfrac{dy}{dx}}{S_o} = \frac{1 - \left(\dfrac{C_o}{C}\right)^2 \left(\dfrac{y_o}{y_c}\right)^3 \left(\dfrac{y_c}{y}\right)^3}{1 - \left(\dfrac{y_c}{y}\right)^3} \quad \text{(38)}$$

one determined the dimensionless ratio $\left(\dfrac{dy}{dx}\right)$: S_o versus $y'_c = \dfrac{y}{y_c}$, with the values for the Chézy factor computed successively from the Manning, Ganguillet-Kutter, and the Prandtl-Kármán (Eq. 31) formulas, and with the friction term $n = 0.013$. (In applying the Ganguillet-Kutter formula, an identical set of values for C, corresponding to $S_o = 0.0004$, was used in all instances. No attempt was made to use the Bazin relation. The latter is well adapted for open-channel flow at higher roughness [$n = 0.020$ to 0.025]. At low roughness, as seen from Fig. 3, the $C = f(R)$-outline is much too flat.)

Computations for $y_c = 2$ ft, 5 ft, and 8 ft also were made for flow in a horizontal channel, using Eq. 11*b* directly. A form for the latter, which would correspond to Eq. 38, is

$$\frac{\dfrac{dy}{dx}}{\dfrac{g}{C^2}} = \frac{\dfrac{dy}{dx}}{\sigma} = \frac{1}{1 - \left(\dfrac{y}{y_c}\right)^3} = \frac{1}{1 - (y'_c)^3} \quad \text{(39)}$$

with σ once more the "critical slope."

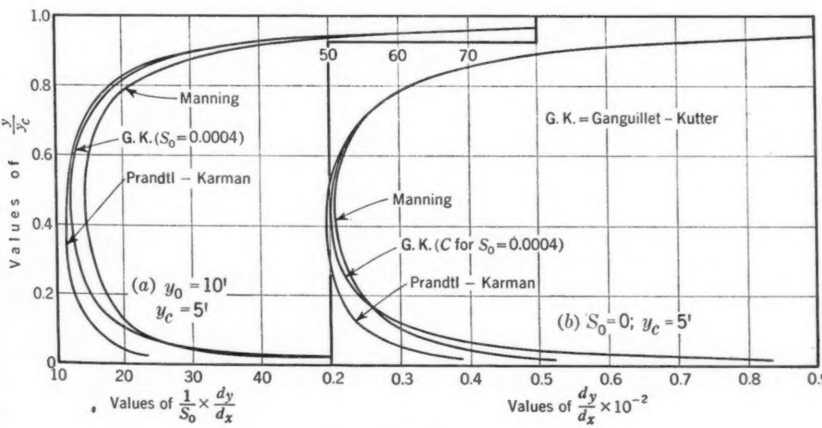


FIG. 6

Curves for the case of $y_c = 5$ ft are presented in Fig. 6. The outlines as shown are typical of the plottings obtained in all instances. All the $\dfrac{dy}{dx}$-curves exhibit only slight change over a comparatively wide middle reach, revealing the previously emphasized flatness of the M_3-curve. The curvature

as derived from Eqs. 38 and 39 becomes pronounced only near $y = y_c$, and close to the bottom, where the computations based on the Bélanger premises of parallel motion with negligible curvature cease to be applicable. Again the curves corresponding to the different formulas vary only in detail, without changing the major features of the picture.

(6) It behooves one now to compare the results of the theoretical computations with actual observations. In practice, M_3 surface curves occur at the foot of spillways and chutes, where the steep incline changes to a mild slope channel, or at the point where water flows from beneath a sluice into a horizontal or mildly sloped bed. In either case the initial surface, in the transition reach, is ipso facto concave. Accordingly, if the initial depth is less than y_i, the subsequent flow must exhibit a portion of convex surface between two concave reaches. Such a succession of forms in practice is highly improbable.

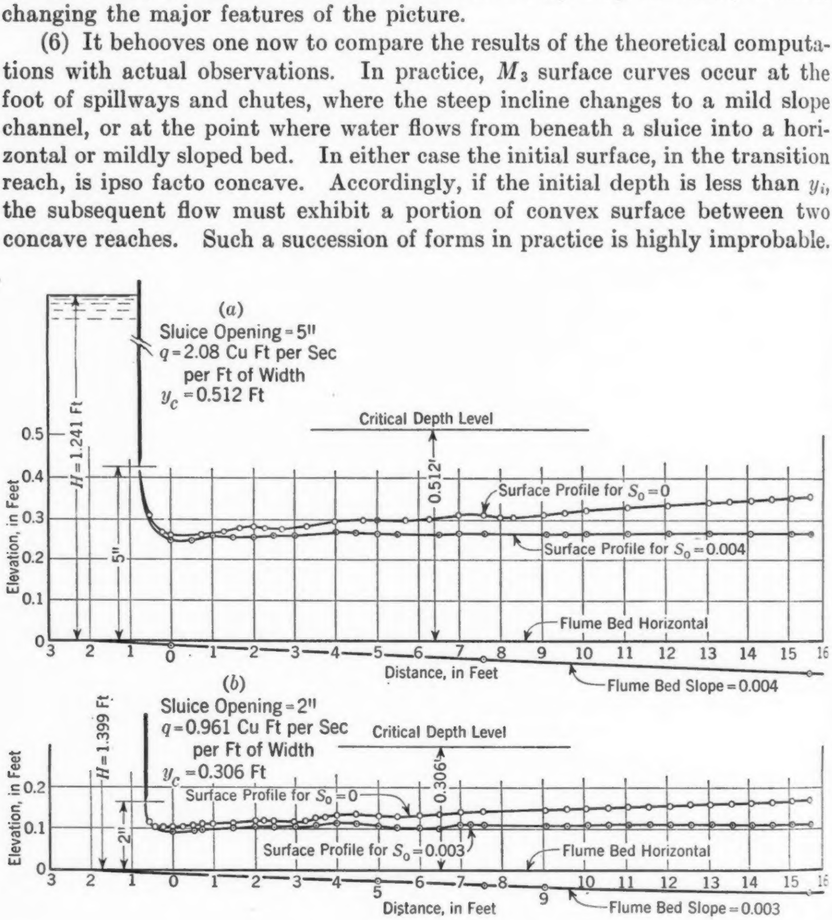


FIG. 7

Moreover, whenever, in hydraulics, a certain equation results in two distinct branches, as in Eqs. 38 and 39 in Fig. 6, one is certain to anticipate that the respective branches reflect altogether different physical regimen, or that flow on one of the branches is unstable and in reality presents wavy features. An example is the double-branched curve[17] which features the height of the jump sustained by flow from under a sluice, and in which the branch for lower initial kinetic energies corresponds to jumps of the "undular" type. Thus, with

[17] "Hydraulics of Open Channels," by Boris A. Bakhmeteff, McGraw-Hill Book Co., Inc., New York, N. Y., 1932, p. 248.

respect to the M_3-curves (Fig. 6), one could reasonably expect that the lower and theoretically convex part of the profile will prove in reality to be more or less undulated. Special experiments in the 6-in. tilting flume of the Fluid Mechanics Laboratory of Columbia University in New York, N. Y., confirm such anticipation. The profiles shown in Fig. 7 are typical of the results obtained for a wider range of sluice openings and initial heads. In each instance, the surface profile in the reach succeeding the initial "contracted" section exhibits distinct undular outlines. The waves are not large, however, and furthermore their size decreases with the increase of the depth, as the curve evolves into the final, practically linear, form. Most characteristically, the effect of waviness was particularly accentuated in the bottom pressures. Apparently, due to the high prevailing velocities, substantial centrifugal effects were exercised even when the curvature, caused by the waves, was comparatively small.

The question arises: Why were not all these facts noticed and recorded in former research? The answer may possibly be found in the general flatness of the M_3-curves, and especially in the fact that profile measurements were taken mostly at considerable intervals. Under such conditions, the relatively small deflections either in one or the other direction from the approximately rectilinear profile could be mistaken easily for erratic experimental deviations. As shown by the curves of Fig. 7, the actual features of the M_3-curves can be revealed only by closely spaced observations. It would be most interesting to have experiments performed on a larger scale and with equipment of sufficient capacity.

In practical computations, the relatively small waviness may be disregarded, and the convex surface which evolves from Eq. 23 for the range of small depths may be considered as an "average" profile.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

HYDRAULIC DESIGN OF DROP STRUCTURES FOR GULLY CONTROL

Discussion

By G. H. Hickox, M. Am. Soc. C. E.

G. H. Hickox,[23] M. Am. Soc. C. E.[23a]—A number of descriptions of stilling basins for large dams have been published, but the results have not been generally applicable. This paper should be of particular interest to the designer of small structures, the costs of which are not sufficient to support independent model studies, and the authors are to be commended on its presentation, and on the studies that preceded it.

It is believed that a word of caution is necessary, however, with respect to the general applicability of the results. The writer does not share with the authors the optimism implied in the following statements (see heading "Application of Drop-Structure Rules to Other Types of Hydraulic Structures"):

> "The principles used in developing the rules for the design of rectangular drop structures for gully control are sufficiently general for application to many problems. There is no great difference between the methods that have been used in insuring energy dissipation and in protecting the structure from undermining and those commonly used in the design of spillways for major engineering structures."

A casual reading of the foregoing statements might lead the reader to assume that the criteria developed in the paper were directly applicable to spillways for major engineering structures, although it is quite probable that the authors did not mean to imply this.

There are at least two important differences between the gully-control structures described in the paper and spillways for large dams. The first of these differences is contained in the statement of the authors (see heading "Development of Design Formulas from Experimental Data: 2. Height of Transverse End Sill")

Note.—This paper by B. T. Morris, Jun. Am. Soc. C. E., and D. C. Johnson, Assoc. M. Am. Soc. C. E., was published in January, 1942, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1942, by John Hedberg, Assoc. M. Am. Soc. C. E.; May, 1942, by Messrs. L. Standish Hall, and J. E. Christiansen; June, 1942, by Walter T. Wilson, Assoc. M. Am. Soc. C. E.; and September, 1942, *Proceedings*, by Messrs. N. A. Christensen and Dwight Gunder, and Boris A. Bakhmeteff and Nicholas V. Feodoroff.

[23] Senior Hydr. Engr., TVA, Hydraulic Laboratory, Norris, Tenn.

[23a] Received by the Secretary August 17, 1942.

"* * * that the loss of kinetic energy incident to the impingement of the nappe on the apron is a large quantity and that this loss increases rapidly with increasing fall height so as to offset, to an appreciable extent, but not completely, the increased energy of the higher falls."

This may be true for the type of structure tested, in which the water falls directly on a horizontal apron, but it is not generally true for major structures. There, an effort is usually made to prevent this type of energy loss because of the tremendous impact forces that would exist immediately at the toe of the structure. Instead of striking on a horizontal floor, the flow is usually turned in a horizontal direction at the toe by means of a vertical curve or bucket having a radius, depending on the depth of flow, of from 35 ft to 100 ft. This gradual change of direction involves less loss of energy and means that a greater amount must be dissipated in the stilling basin. This difference is not on the side of safety.

The second important difference is in the assumption made with respect to tailwater elevations. The tests apparently were made on the assumption that flow below the structure would occur at relatively high velocities and small depths. Thus, the ground roller below the end sill was always formed under similar conditions. Dams on large rivers are usually relatively low, and the elevation of tailwater becomes an important factor. In many cases, tailwater for maximum discharge may be above the spillway crest. This is true at Dams Nos. D1, D2, and D3, Table 5, and at two other main dams on the Tennessee River, and doubtless in many other instances. When the tailwater is sufficiently high above the spillway crest, the type of flow changes and, instead of plunging beneath the tailwater to the apron on the river bed, continues near the surface for a time. The conditions for energy dissipation are thus completely changed and the criteria proposed are no longer applicable.

TABLE 5.—Comparison of Actual Dam Dimensions with Eqs. 6*a* and 7*c*

Key designation	h (ft)	d_c (ft)	L (Ft)		h' (Ft)	
			Actual	By Eq. 6*a*	Actual	By Eq. 7*c*
D1	38	30.7	130	129	5	15.4
D2	20	30.7	106	167	8	15.4
D3	48	24.5	150	108	5	12.2
D4	56	24.5	167	113	5	12.2
D5	237.5	17.3	420	165	17	8.6
D6	203.5	24.5	395	186	12	12.2

The criterion for the length of structure is based on the assumption that the discharge falls freely from the crest to the apron. This is not true for overflow spillways on gravity dams because the requirements of stability place the toe of the dam downstream from the normal trajectory of the overfalling sheet. As a result, the distance L from crest to end sill is necessarily increased. Table 5 is a comparison of values of L and h' as calculated from Eqs. 6*a* and 7*c* and as adopted on the basis of model tests for a number of dams built by the Tennessee Valley Authority. These dams were all built with horizontal aprons and plain end sills, and provide typical examples of both low-head and high-

head dams to which a general relationship might be expected to apply. The lack of satisfactory agreement is evident.

Another factor of importance in many spillway designs is the formation of waves below the spillway and their effect on bank erosion and navigation. The slope of the upstream face of the end sill has an important bearing on the production of waves as well as on the formation of the ground roller. On none of the dams listed in Table 5 is the upstream face of the end sill vertical. Until many more experiments have been made under a wide range of conditions, special investigations should be made for all major structures.

The writer is puzzled by criterion (1), which states (see heading "Criteria for Satisfactory Drop-Structure Performance"):

> "The kinetic energy of the effluent stream, as measured by the excess of local velocity over the velocity for absolute minimum specific energy, shall be a minimum."

The reason for the choice of the kinetic energy at the critical velocity as a reference base is not clear, since it is the total energy, and not the kinetic energy, that is a minimum at that point. It would also seem necessary to specify the point at which the energy (or velocity) was measured, since there is considerable dissipation of energy a short distance below the structure, as illustrated by Fig. 6. If the purpose of the structure is the prevention of erosion, is not a low bottom velocity the end to be desired?

It is noted that extensive use was made of photography for taking data. The possibilities of photography for this purpose are very great but, so far, comparatively little progress in its use as a laboratory tool has been reported. The authors are to be commended for their use of this versatile tool.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

EFFECT OF VARIATION OF ELASTIC CHARACTERISTICS ON STATIC UNKNOWNS

Discussion

By JOSEPH A. WISE, M. AM. SOC. C. E.

JOSEPH A. WISE,[7] M. AM. SOC. C. E.[7a]—The fundamental principles of the effect of the variation of elastic characteristics and their application to specific structures are well presented in this excellent paper. In the analysis given by the author, a statically determinate basic structure is taken, and the n redundant stresses are superimposed upon it. If, instead, a statically indeterminate basic structure is used (one having $n - 1$ redundants, such as is used in obtaining an influence line for the nth redundant), the computations are simplified and some new and interesting relationships appear. In effect, this reduces all structures to singly redundant types, assuming that the remaining redundants have been determined. To avoid introducing new and possibly confusing nomenclature, the symbols used in Professor Hrennikoff's paper will be adhered to, but with the understanding that the moments, M_o, refer to moments due to the given loading in the $(n - 1)$-degree statically indeterminate base structure, and the moments m_a are those in the same base structure due to a unit load applied in place of the nth redundant. Thus, the only difference from a singly redundant structure is that the moments M_o and m_a must be calculated for an $(n - 1)$-degree statically indeterminate structure instead of being determined by statics alone.

The analysis first will be made for structures subjected to bending alone. Then, in general:

$$X_a = -\frac{\delta_{ao}}{\delta_{aa}} \quad \text{.......................(44)}$$

$$dX_a = -\frac{1}{\delta_{aa}}\left(d\delta_{ao} - \frac{\delta_{ao}}{\delta_{aa}}\, d\delta_{aa}\right) \quad \text{..............(20a)}$$

NOTE.—This paper by A. Hrennikoff, Assoc. M. Am. Soc. C. E., was published in January, 1942, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1942, by A. H. Finlay, Assoc. M. Am. Soc. C. E.; and May, 1942, by Stefan J. Fraenkel, Jun. Am. Soc. C. E.

[7] Lt.-Comdr. (CEC), U.S.N.R., Electrical Eng. Dept., U. S. Naval Academy, Annapolis, Md.; and Assoc. Prof. of Structural Eng., Univ. of Minnesota, Minneapolis, Minn. (on leave).

[7a] Received by the Secretary September 9, 1942.

and

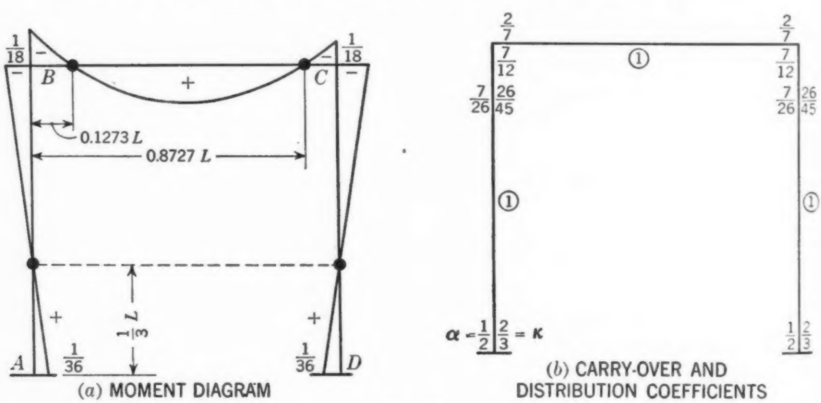
$$dX_a\,(\max) = \frac{i}{\delta_{aa}} \int \left| \left(\frac{M_o\, m_a}{E\,I} + X_a \frac{m^2{}_a}{E\,I} \right) \right| ds \quad \text{(20b)}$$

Eq. 20*b* also can be written:

$$dX_a\,(\max) = \frac{i}{\delta_{aa}} \int \left| \frac{(M_o + X_a\, m_a)\, m_a}{E\,I} \right| ds = \frac{i}{\delta_{aa}} \int \left| \frac{M\, m_a\, ds}{E\,I} \right| \quad (45)$$

in which M is the final or true moment at any segment, ds, and is equal to $M_o + X_a\, m_a$. Eq. 45 shows that dX_a will have a zero value (that is, a change in $E\,I$ in a segment ds will not change X_a) when either M or m_a is zero. In integrating along the members of the structure, whenever either M or m_a passes through zero the integrand changes sign; hence, the segments between these null points are alternately increased and decreased in $E\,I$. If M and m_a should happen to have a common null point, this null point is omitted, since both change sign in passing through it. Thus, a general rule could be formulated as follows:

> If it is desired to find the maximum variation dX_a of the moment at a point in a structure, determine the actual moments M at all points. Then consider the redundant X_a to be replaced by a pair of unit couples acting in the same sense, and determine the moments m_a in the otherwise unloaded statically indeterminate base structure. Determine the null points; then compute dX_a from Eq. 45, integrating between singular null points.



(*a*) MOMENT DIAGRAM

(*b*) CARRY-OVER AND DISTRIBUTION COEFFICIENTS

FIG. 10

This analysis can be readily extended to structures with axially stressed members, as indicated by Professor Hrennikoff, so that Eq. 45 becomes:

$$dX_a = \frac{i}{\delta_{aa}} \sum \left| \frac{N\, n_a\, L}{E\,A} \right| \quad (46)$$

It is obvious that members having N and n_a of like sign must be increased in area, and the other members must be decreased in area (or in $E\,A$) to obtain the greatest increase, dX_a.

To illustrate the application of the foregoing principle, the frame of Fig. 1 will be analyzed. The writer has used a precise moment-distribution method[8] in the analysis, as it gives accurate results with rapidity, but any method could be used to obtain the same results. Fig. 10 shows the actual moments M at ends of all members, and the moment diagram. The null points determined by this method are shown by heavy dots. The moment at point B is expressed by

$$|M_B| = \frac{w\,L^2}{12} \times \frac{3 \times \frac{2}{7}\left(1 - \frac{2}{7}\right)}{1 - \frac{2}{7} \times \frac{2}{7}} = \frac{w\,L^2}{18} \quad \text{.............(47)}$$

Fig. 11 shows the analysis for m_a. Here the moment at point B is taken as the redundant X_a and the frame is analyzed for unit couples applied as shown.

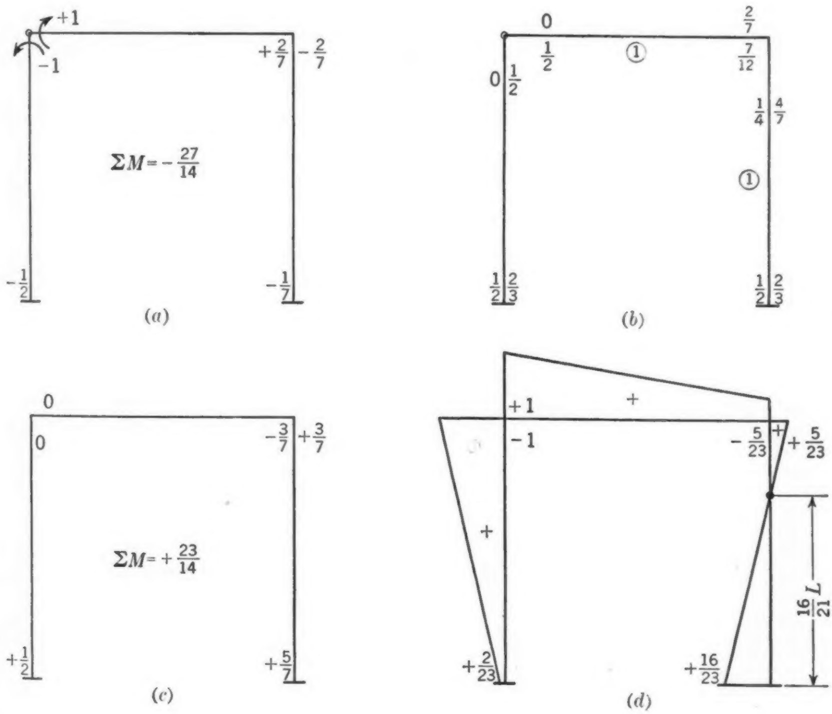


FIG. 11

Fig. 11(a) gives the first moment distribution, assuming no sidesway; Fig. 11(b) gives the precise carry-over and distribution coefficients; Fig. 11(c) gives moments due to an assumed sidesway; and Fig. 11(d) gives moments corrected for

[8] "A Precise Moment Distribution Method," by Joseph A. Wise, *Journal*, A.C.I., November, 1938, p. 93.

sidesway. The moment at point A, for example, is $-\frac{1}{2}+\frac{27}{23}\times\frac{1}{2}=+\frac{2}{23}\cdot$ Fig. 11 locates another null point. These null points agree substantially with those determined in the paper.

If the equations for M and m_a are written for the three parts of the frame:

Left leg—

$$M = \frac{w\,L^2}{36}\left(1 - 3\,\frac{y}{L}\right)\ldots\ldots(48a)$$

$$m_a = \frac{1}{23}\left(2 + 21\,\frac{y}{L}\right)\ldots\ldots(48b)$$

$$M\,m_a = \frac{w\,L^2}{828}\left(2 + 15\,\frac{y}{L} - 63\,\frac{y^2}{L^2}\right)\ldots\ldots(48c)$$

Beam—

$$M = \frac{w\,L^2}{18}\left(9\,\frac{x}{L} - 9\,\frac{x^2}{L^2} - 1\right)\ldots\ldots(49a)$$

$$m_a = -\frac{1}{23}\left(23 - 18\,\frac{x}{L}\right)\ldots\ldots(49b)$$

$$M\,m_a = \frac{w\,L^2}{414}\left(23 - 225\,\frac{x}{L} + 369\,\frac{x^2}{L^2} - 162\,\frac{x^3}{L^3}\right)\ldots\ldots(49c)$$

Right leg—

$$M = \frac{w\,L^2}{36}\left(1 - 3\,\frac{y}{L}\right)\ldots\ldots(50a)$$

$$m_a = \frac{1}{23}\left(21\,\frac{y}{L} - 16\right)\ldots\ldots(50b)$$

$$M\,m_a = \frac{w\,L^2}{828}\left(-16 + 69\,\frac{y}{L} - 63\,\frac{y^2}{L^2}\right)\ldots\ldots(50c)$$

and

$$\delta_{aa} = \int\frac{m^2{}_a\,ds}{E\,I} = \frac{1}{529}\left[\int_0^L\left(2 + 21\,\frac{y}{L}\right)^2 dy + \int_0^L\left(23 - 18\,\frac{x}{L}\right)^2 dx + \int_0^L\left(21\,\frac{y}{L} - 16\right)^2 dy\right] = \frac{21\,L}{23\,E\,I}\ldots(51)$$

Substituting in Eq. 45,

$$dX_a = \frac{w\,L^2\,i}{756}\left[\int_0^L\left|2 + \frac{15\,y}{L} - \frac{63\,y^2}{L^2}\right| d\left(\frac{y}{L}\right) + 2\int_0^L\left|-23 + \frac{225\,x}{L} - \frac{369\,x^2}{L^2} + \frac{162\,x^3}{L^3}\right| d\left(\frac{x}{L}\right) + \int_0^L\left|-16 + \frac{69\,y}{L} - \frac{63\,y^2}{L^2}\right| d\left(\frac{y}{L}\right)\right]\ldots\ldots(52)$$

This agrees exactly with Eq. 30*a*. However, the null points have already been found, and it is only necessary to evaluate Eq. 52 between these null points. The result will be the same as in the paper.

From a study of Eq. 45 it will be seen that dX_a will be most affected by changes in $E\,I$ where the product $M\,m_a$ is a maximum and, by obtaining the maximum of each of the three terms in Eq. 52, it will be found that the most sensitive points in the frame are: Left leg, $\frac{y}{L} = \frac{5}{42}$; beam, $\frac{x}{L} = 0.386$; and right leg, $\frac{y}{L} = \frac{23}{42}$. Thus, a curve $M\,m_a$ could reveal the parts of the structure in which changes in $E\,I$ have the greatest effect on X_a. In the demolition of structures it might be quite useful to have this information. Also, if the $M\,m_a$-curve is plotted, a planimeter could be used to obtain the mechanical integration of Eq. 45.

The approximate integration by finite summation can be easily applied to this analysis. The frame of Fig. 5 is analyzed, as in the paper, by substituting the numerical values from Table 2:

$$dH = \frac{5{,}004}{1{,}201}\,w\,i = 4.17\,w\,i \qquad (53)$$

In comparing the moment at the crown and the moment at the knee of this frame, it is noted that the two m_a-diagrams are the same and, since M is obviously the same for both cases, the variation of dX_a will be the same for both. However, in obtaining K, dX_a is divided by X_a for each of the respective cases, and for the moment at the crown, $|X_a| = 63.86$, whereas, for the moment at the knee, $|X_a| = 248.64$. Therefore, the K-values for these two points should be inversely proportional to the respective moments, or K (knee) is to K (crown) as 63.86 is to 248.64. The respective values for K—0.263 and 1.024—given in the paper agree with this ratio. From this it can be concluded that K tends to be largest where stresses are smallest. In fact, if one were to investigate a point of inflection in the structure, where X_a is zero, one would find K becoming infinite, as it obviously should, since any change in $E\,I$ elsewhere in the structure would result in a moment at this point where no moment had existed previously. Thus, it appears that K is not entirely satisfactory as a measure of the sensitivity of the structure to changes in the elastic constants. The designer is not much concerned about changes in moment where the moments are small and the stresses are far below the allowable working stresses. It is

TABLE 2.—ALTERNATE SOLUTION FOR dH (MAX), FIG. 4

Segment	$\frac{M}{w}$	m_a	I	$\left\lvert\frac{M\,m_a\,\Delta s}{I}\right\rvert$	$\frac{m^2\,\Delta s}{I}$
1	− 31.08	2	9.6	26	2
2	− 93.24	6	13.3	168	11
3	−155.40	10	18.0	345	22
4	−217.56	14	23.7	514	33
5	−189.27	16	20.8	703	62
6	− 89.27	16	12.3	580	104
7	− 14.27	16	7.1	161	180
8	+ 35.73	16	4.16	687	308
9	+ 60.73	16	2.67	1,820	479
Total	....	..	....	5,004	1,201

suggested that a factor

$$K' = K \frac{f_a}{f_w} \qquad (54)$$

be used as a measure of sensitivity, in which f_a is the actual maximum fiber stress in the structure at the location of the redundant and f_w is the allowable working stress. At points of inflection K' would be indeterminate, being of the form $\infty \times 0$, and would not be considered as significant.

The suggestion made by Professor Hrennikoff that sensitivity appears to increase as the number of static unknowns increase can be explained at least partly by the fact that, in general, as the redundancy of the structure increases, its stiffness increases, so that the deflections due to unit couples, δ_{aa}, decrease. Since this term appears in the denominator of Eq. 45, a decrease in it will cause an increase in dX_a.

The opinions or assertions contained herein are the private ones of the writer and are not to be construed as official or reflecting the views of the Navy Department or the naval service at large.

Corrections for *Transactions:* In January, 1942, *Proceedings*, page 85, change "moment of inertia" (line 23) to "modulus of elasticity"; change Eq. 37*c* to read "dH (max) $= 4.18\, w\, i = 0.269\, i\,|H|$"; in Eq. 37*d* change "65.39" to "66.91"; on page 90 (line 1) change "1.024" to "1.048"; on page 90 (line 2) change "0.263" to "0.269"; and in Fig. 6 change "0.263," "0.263," and "1.024" to "0.269," "0.269," and "1.048," respectively. See also erratum on page 664 of April, 1942, *Proceedings*.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

STRESS CONCENTRATIONS IN PLATES LOADED OVER SMALL AREAS

Discussion

By Messrs. L. A. MacColl, and L. W. Teller and Earl C. Sutherland

---

L. A. MacColl,[36] Esq.[36a]—The paper by Dean Westergaard makes a valuable contribution to the practical solution of certain problems which are of great importance. It undoubtedly will be welcomed by all engineers who have occasion to work with the subject.

Briefly, the idea is that of expressing approximately such quantities as bending moments, twisting moments, etc., resulting from the application of a moderately concentrated load to an elastic plate, as sums of small numbers of terms, which severally have simple interpretations in terms of the location and distribution of the load.

To the writer the names "place coefficients" and "area coefficients" seem to be unfortunately chosen, for an inspection of the basic formulas shows that they are not coefficients in any very usual sense of the word. By Eq. 13, B, K, C, and S—or, more exactly, $\frac{(1+\mu)\,B}{4\,\pi}$, $\frac{(1+\mu)\,K}{4\,\pi}$, $\frac{(1-\mu)\,C}{8\,\pi}$, and $\frac{(1-\mu)\,S}{8\,\pi}$—are bending moments per unit load. Likewise, by Eq. 14, $\frac{(1-\mu)\,D}{8\,\pi}$ and $\frac{(1-\mu)\,T}{8\,\pi}$ are twisting moments per unit load. Hence, the writer would prefer some such names as "place moments" and "area moments," or "location moments" and "distribution moments."

It is suggested in the paper that the so-called area coefficients, K, S, and T, may be determined more easily from the partial differential equations 24 and 25, than from the explicit formulas, Eqs. 11 and 12. The writer doubts that this is true under any very general conditions. Whatever simplification the partial differential equations have afforded in the particular cases con-

---

Note.—This paper by H. M. Westergaard, M. Am. Soc. C. E., was published in April, 1942, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1942, by Messrs. George R. Rich, Dana Young, and A. Nádai.

[36] Bell Telephone Laboratories, Inc., New York, N. Y.

[36a] Received by the Secretary August 17, 1942.

sidered in the paper has resulted, he thinks, from the fact that those cases are very simple and special, and are not at all typical of the general situation. Certainly, a set of explicit formulas, comparable to Eqs. 11 and 12, is the goal of any method for solving differential equations; and it should be considered fortunate that in the present case the formulas are so simple and so easily interpreted.

L. W. TELLER,[37] ASSOC. M. AM. SOC. C. E., AND EARL C. SUTHERLAND,[38] ESQ.[38a]—That portion of this very useful paper which deals with the action of a large slab on an elastic subgrade is of particular interest to the highway engineer and it is to this particular part of the paper that this discussion will be limited.

In his original analysis of the load-stress relations in a concrete pavement slab,[39] Dean Westergaard assumed that the load was applied to the surface of the pavement over a uniformly loaded small circular area in the cases of (1) a corner loading and (2) a load applied at an interior point some distance from the slab edges and over a uniformly loaded semicircular area for the case of a load applied at an edge at some distance from a corner. For these conditions of loading, formulas for the calculation of the critical load stresses were presented.

In the paper under discussion, the early analysis is extended to permit the computation not only of the critical load stresses but also of other load stresses in the vicinity of the loaded area both for areas that are circular in shape and for areas other than circular in shape. Of the latter, the elliptical area, such as that used by Dean Westergaard in the numerical example (Table 2), is of particular interest since it is a shape which approximates quite closely that of the area of contact of a large pneumatic tire at normal inflation and capacity load.

A number of years ago the Public Roads Administration began a study of the structural behavior of concrete pavement slabs at the Arlington Experiment Farm, Virginia. A part of this investigation was an experimental study of the load-stress relations in pavement slabs of uniform thickness, being, in effect, a comparison of the behavior as predicted by the Westergaard theory with the behavior as actually observed on pavement slabs of ordinary size. Four reports of this work have been published[40] and the fifth (and final) report, which concerns the Westergaard analysis most directly, is nearing completion (August, 1942). The reports already published describe the scope of the investigation and the general procedures and apparatus used.

One of the questions left unanswered in the original analysis by the author was that of the effect of the shape of the loaded area on the magnitude of the

---

[37] Senior Engr. of Tests, Public Roads Administration, Federal Works Agency, Washington, D. C.

[38] Highway Engr., Public Roads Administration, Federal Works Agency, Washington, D. C.

[38a] Received by the Secretary August 29, 1942.

[39] "Computation of Stresses in Concrete Roads," by H. M. Westergaard, *Proceedings*, Fifth Annual Meeting of the Highway Research Board, 1926; also *Public Roads*, Vol. 7, No. 2, April, 1926.

[40] "The Structural Design of Concrete Pavements," reported by L. W. Teller and Earl C. Sutherland.

Part 1—"A Description of the Investigation," *Public Roads*, Vol. 16, No. 8, October, 1935.
Part 2—"Observed Effects of Variations in Temperature and Moisture on the Size, Shape and Stress Resistance of Concrete Pavement Slabs," *ibid.*, No. 9, November, 1935.
Part 3—"A Study of Concrete Pavement Cross Sections," *ibid.*, No. 10, December, 1935.
Part 4—"A Study of the Structural Action of Several Types of Transverse and Longitudinal Joint Designs," *ibid.*, Vol. 17, Nos. 7 and 8, September and October, 1936.

critical stress from a given wheel load. So long as it was unanswered it has been the subject of considerable discussion. In an effort to obtain some data on this particular question, a limited program of load tests on areas simulating tire contact areas was made in connection with the aforementioned general investigation.[40] Since these tests concern one of the problems analyzed in the present paper, it is thought that the experimental data may be of interest.

The tests were made on a concrete pavement slab of normal size and of 8-in. uniform thickness. This pavement was laid on a stabilized subgrade formed by incorporating sand with the original subgrade material and compacting to maximum density at the optimum moisture content. Loads were applied on bearing plates cut to a shape which simulated the area of contact of low pressure pneumatic tires. This shape is shown in Fig. 3(*a*). It is a

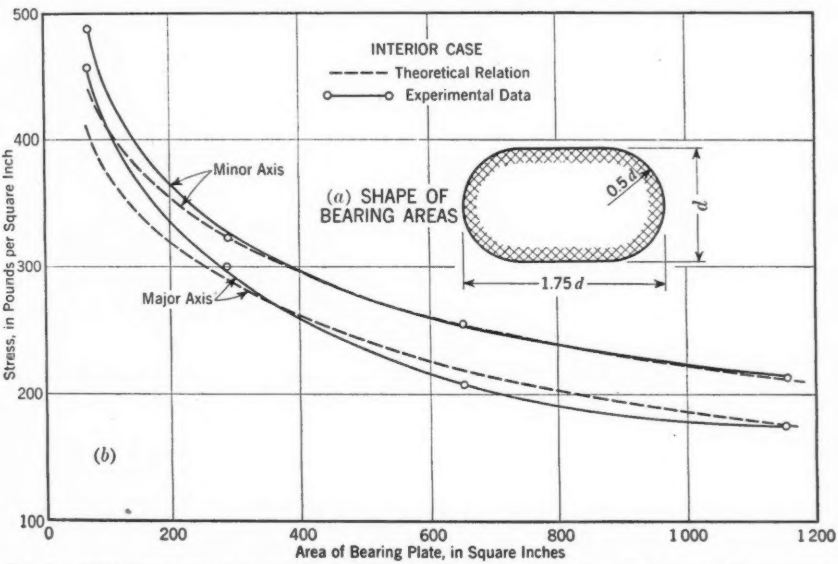


FIG. 3.—COMPARISON OF COMPUTED STRESSES WITH THOSE DETERMINED BY STRAIN MEASUREMENT ALONG BOTH MAJOR AND MINOR AXES OF ELLIPTICAL LOADED AREAS

conventionalized average of a fairly large number of actual tire impressions and, even when extended to the larger sizes, seems to conform rather well to available tire imprints. In these comparisons the impressions of airplane tires at recommended capacity load were used.

In the loading tests four bearing plates of this conventionalized shape of the sizes were used, as follows:

Width (minor axis, *d*) in inches	Length (major axis) in inches	Area, in square inches
6.86	12.00	72
13.71	24.00	289
20.57	36.00	649
27.43	48.00	1,155

The loading apparatus was arranged so as to distribute the applied load uniformly over the area of contact of the bearing plate. Strains were measured in the upper surface of the pavement at the center of the bearing area along both the major and minor axes of the area. The gage used was the recording gage described in the first of the published reports mentioned previously.[39]

The modulus of elasticity of the concrete E equals 4,800,000 lb per sq in., and the modulus of subgrade reaction k equals 250 lb per cu in., as determined by deflection tests on the pavement slab in the manner suggested by Dean Westergaard in one of his earlier papers.[41]

Fig. 3(*b*) shows a comparison between the observed stresses in the aforementioned 8-in. pavement slab and corresponding stresses computed by the methods described in the paper. The comparison is made for both the major and minor axes for each of the four sizes of loaded area that were used in the experimental work. The wheel-load $P = 20{,}000$ lb; Poisson's ratio $\mu = 0.15$; and the corrected combined coefficient[39] $(B + K)'$ was used.

It will be noted that, in general, there is a remarkable agreement between the theoretical and experimental values. The relative differences observed between the stresses along the major and minor axes of a given area are almost exactly those which the theory indicates should exist. The magnitudes of the stresses for the conditions selected for comparison are in close agreement except for the smallest bearing area where the values determined by measurement are about 10% greater than the corresponding computed values. This affects the shape of the curves showing the relation between the size of loaded area and stress as is evident in Fig. 3(*b*). However, even with this exception included, the observed effect of size of loaded area on stress agrees very well with that expressed in the theoretical treatment.

Another comparison which is of interest concerns the numerical example given in Table 2 of the paper. For the conditions assumed in this example, it is indicated that a dual tire with a load of 5,000 lb per tire (or a wheel load of 10,000 lb) will cause a maximum pavement stress which is only 1.5 times that caused by a single tire with a load of 5,000 lb. Thus it is indicated that by using dual tires of suitable size the wheel load can be doubled with only a 50% increase in the corresponding critical pavement stress.

Another interesting point shown by the computed stress values of Table 2 is that, for the case of the dual tire in this comparison, the critical stress (σ_y) has a practically constant value from the center of the area of contact of one tire to the center of the area of contact of the other tire.

Several years ago the Public Roads Administration made a limited series of load tests in which an effort was made to compare the stress produced by a given load when the load was applied through a single circular bearing plate with the stress developed when twice that load was applied through two plates of the same size, separated by a distance comparable to the dual tire spacings of large tires. Although the program was quite limited and the data apply

41 "Analytical Tools for Judging Results of Structural Tests of Concrete Pavements," by H. M. Westergaard, *Public Roads*, Vol. 14, No. 10, December, 1933.

only to circular areas of contact, the tests afford an interesting comparison with the indications of Table 2.

In the tests referred to, loads were applied on an 8-in. circular plate in the interior region of a slab of uniform thickness. Strains were measured in the direction of the longitudinal axis of the pavement in the center of the loaded area and at various distances from the center on a line perpendicular to the longitudinal axis of the pavement through the center of the loaded area. From these strain data a curve was developed showing how the magnitude of the longitudinal stress varied across the pavement slab as the distance from the loaded area was changed.

With this stress-variation curve as a basis, the effect of two loads was examined by combining two such curves displaced horizontally 12 in. with respect to each other as shown in Fig. 4(*a*) and obtaining combined stress

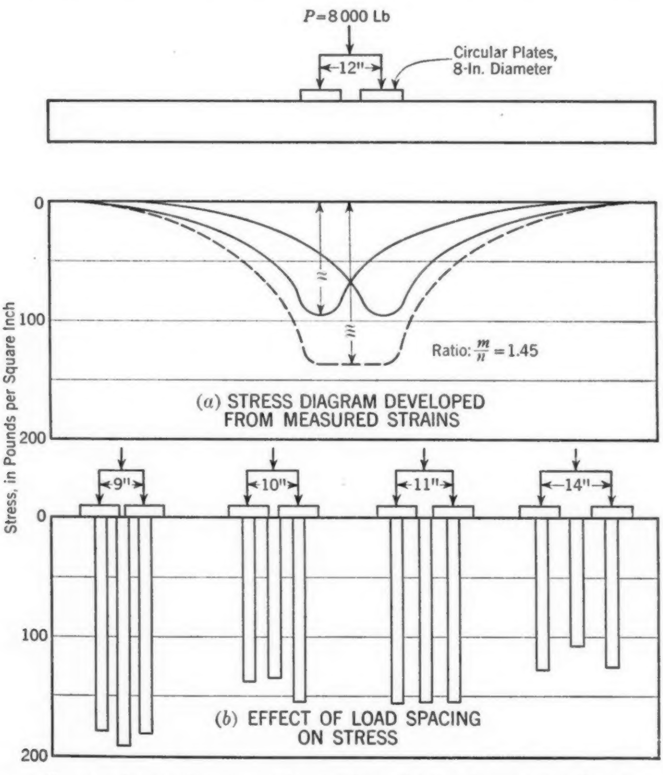


FIG. 4.—STRESS CONDITIONS UNDER DUAL LOADS; INTERIOR OF SLAB

values by addition. This study indicated that a load of 8,000 lb, applied through dual 8-in. diameter areas, would cause a stress 1.45 times as great as would be caused by 4,000 lb applied through a single area of the same size.

Fig. 4(*b*) shows some additional data obtained in the same experiments by tests in which strains were measured under and between the loaded areas.

It will be noted that both the combined stress curve of Fig. 4(*a*) and the data for the 11-in. spacing of Fig. 4(*b*) indicate that the stress between the loaded areas is practically equal to that within the loaded areas.

Although the data presented in this discussion are quite limited in scope and apply to but one part of the paper under discussion, the close agreement between the behavior predicted by the theory and that observed during the loading tests tends to inspire confidence in the general soundness of the theoretical treatment.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

RELATIVE ANGULAR, LINEAR, AND TRAVERSE ACCURACIES IN CITY SURVEYS

Discussion

By Messrs. Cleveland B. Coe, W. H. Rayner, Philip Kissam, and George D. Whitmore, C. C. Miner and W. O. Byrd

Cleveland B. Coe,[2] M. Am. Soc. C. E.[2a]—The author of this paper joins a small and select group—those who offer something for the advancement of the professional knowledge and ability of the surveyor. In any issue of almost any technical publication, structural, hydraulic, and sanitary engineers may find results of investigation and analysis which will bear more or less upon their daily problems, but the land surveyor is the "forgotten man" of the profession, although not of the general public, to whom he is the "engineer."

Truly, the phrase used by Mr. Bauer is a masterly one (see "Synopsis"): "Under the conditions and with the equipment that the economics of the profession force upon him." The economics of the profession have forced considerable shoddy practice on many men who knew what should be done to give reliable results to their clients. The clients neither know nor care how the surveyor arrives at his results; nor are they interested in the accuracy attained, in many cases.

The writer surveyed farm lands for many years, in competition with day labor of the crudest type. Very frequently he went on a job alone, and assistance was provided by the client, in the form of his sons and nephews. This was, and is, a condition of very common occurrence and will continue as long as registers of deeds are permitted by law to accept the descriptions and plats made by unlicensed surveyors. Much has been done to correct this condition, but much educational work remains before the landowner becomes willing to pay the price which efficient help and accurate measurement of land should reasonably cost.

The paper deals entirely with city surveys and concerns itself with property of considerable value. The writer's experience was with mountain and farm

Note.—This paper by S. A. Bauer, Assoc. M. Am. Soc. C. E., was published in May, 1942, *Proceedings*.

[2] Lt.-Col., Corps of Engrs. U. S. Army, Knoxville, Tenn.

[2a] Received by the Secretary July 17, 1942.

land, worth from $10 to $300 per acre, and the conditions and equipment forced upon him by the economics of the profession were certainly inadequate, even for so low a valuation. If similar studies are presented, as hoped by the author, perhaps someone will present the rural point of view.

The survey of farm lands by the private surveyor, as distinguished from cadastral surveys of large areas by government or corporation surveyors, is generally in one of the following categories:

(1) To establish a dividing line or a disputed corner;
(2) To ascertain true acreage; or
(3) To subdivide property for inheritance or sales purposes, either in large tracts or small lots.

Case (1), in the writer's experience, comes more under the head of an art than a science. In Tennessee, there are no applications of the law of township and section, but all property traces back to original grants, mixed up in the wildest confusion. In no case has the procedure been clear-cut and capable of solution entirely by scientific methods and careful application of angular and linear measurement. Diplomacy is an important tool of the surveyor, and practically all final results are the result of compromise, either known or unknown to the parties to the dispute. Under such conditions, repeating angles and tension in a tape play no part.

Case (2) requires closer adherence to the rules and is the one in which old notes can be reviewed and rerun. Unfortunately for such studies, it is very seldom that a corner monument can be exactly occupied. Nearly all lines are offset lines, with temporary monuments, tied to the corners by side shots. Here the parsimony of the owner plays a great part, not only by preventing the employment of skilled help but by making it necessary to run the offsets in places requiring the least clearing, not following the boundary as closely as it should be. Here, also, the rural objection to measurement in a horizontal plane manifests itself, not infrequently making it necessary to measure vertical angles by any means at hand and to correct slope measurements in the office without the knowledge of the client.

Case (3) is an extension of case (2), with the boundary survey checked by numerous crosslines. In this case, a much closer adjustment is possible.

Unfortunately, the writer's notes are stored "for the duration." It would be of interest for some country surveyor to produce a tabulation of his work, showing the accuracies attained under fire (sometimes literally), as set forth against the value of the land surveyed. All credit is due the many honest and efficient men in all parts of the United States who give reasonable results under very unfavorable conditions and thereby aid in the upbuilding of the profession, which has been sadly disgraced at different times and places by men who are neither efficient nor honest.

W. H. RAYNER,[3] Assoc. M. Am. Soc. C. E.[3a]—Careful accounts of surveying procedures, and analyses of the results obtained, are meager and it is gratifying to have such a report by Mr. Bauer.

[3] Associate Prof., Civ. Eng., Univ. of Illinois, Urbana, Ill.

[3a] Received by the Secretary July 29, 1942.

It well exemplifies the fact that, by proper attention to the more important sources of error, the accuracy of surveys, using ordinary equipment, and at small additional expense, can be greatly increased over the crude results so often obtained.

There are no very definite criteria for comparing the merits of the different surveying instruments, and it has become customary to use the least count of the vernier to designate the character of the instrument used. This is unfortunate because there are wide differences between transits having the same vernier count; also, many transits having 1′ verniers are superior to others with 30″ verniers.[4] The better instruments yield better results with such a saving in time that it is poor economy to use any but the best quality on surveys of any considerable extent.

The use of the "probable error" for two observations as computed by Eqs. 2 and 4, and as described under the heading "Tabulations of Linear Readings," is scarcely justified. This quantity is based on the assumption that a normal distribution of the accidental errors (residuals) of a series of observations is under investigation, and such a distribution can be obtained only where a considerable number of observations are used. It is certainly dubious practice to apply the formulas to a series of less than ten observations.

It is stated that "The probable error in a single reading of a 1-min transit is $\pm$ 25" (see heading "Findings of Fact from Tabulations," Item (*f*)), but it is not clear under just what conditions this value was found. In particular, an important factor is the reading lens that was used. The writer found in his investigations[4] that, in comparing the results of reading angles with and without a reading lens, the probable error of single readings with the naked eye was reduced about one half when a three-diameter lens was used. It was also found that a large part of the error of single readings of angles is caused by the inability of the observer to set the vernier precisely at zero. This condition explains, in part, why the errors of reading angles with ordinary equipment are so greatly reduced when the method of repetitions is used.

The analysis of results evidently has not differentiated sufficiently between the effects of accidental and systematic errors. For example, all traverses were run as loops, wherein the effects of systematic errors do not become evident. Thus, if a tape were of incorrect length, all sides would be affected proportionately, and the calculated error of closure would in no way be affected by this condition. Also, a constant error in the measurement of the temperature would produce a like effect. Further, the duplicate measurement of a line under the same conditions will not reveal the presence of any systematic errors. This fact probably accounts for the fact that the average accuracy of lines measured in different years was less than that for lines measured forward and back on the same day (see Fig. 3).

Accidental errors are compensative in their effects, and hence the errors as expressed in this report should be smaller for a large number of observations than for a few. This is exemplified by the fact that the accuracy found in taping long lines was greater than for short lines (curves 6 and 7, Fig. 3). This

[4] "Two Investigations on Transit Instruments," by W. H. Rayner, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill., 1938.

latter consideration is not verified, however, in the case of traverse closures, for no significant difference was found for traverses of different lengths. This unusual result seems to call for some explanation. It is very probable that the computed errors of closure would be larger if the traverses had closed upon each other, instead of being run as closed loops.

It would seem that, where so much care was used in measuring the distances, reference would be made to the U. S. Bureau of Standards to determine the true lengths of the tapes used. The fact that two tapes are of the same length is not complete evidence that they are correct. Their true length is never known until they have been compared with the United States standard, or with a "standardized" tape which has thus been compared.

The effects of the combined errors in distance and angle measurements in traversing are scarcely as simple as that indicated by Eq. 5. The writer attempted an analysis[5] of these effects in 1928. The relations between the linear and angular errors are complex and, in general, their effects must augment each other, so that the total error of closure is larger than the effect of either one taken separately.

Considering the foregoing remarks, the following comments may be made about the conclusions offered by the author:

(*a*) The analysis in the sixth paragraph preceding Eq. 5 leads to conclusion (2)—namely, that "Retaping a line, forward and back, under average conditions, adds nothing to actual accuracy." This analysis is faulty in that it disregards the effects of systematic errors that would appear in measurements made at different times but not in repeated measurements made under the same conditions. In other words, it is probable that the true errors in the measurements made on the same day are larger than the errors indicated by the duplicate measurements. If so, conclusion (2) cannot be justified by the evidence and analysis used. Clearly, as regards the accidental errors involved, the error of the mean of two measurements will be less than that for a single measurement.

(*b*) Conclusion (3) is based on the oversimplified analysis of the relations of the various errors in traversing. It is conceivable that, in a given traverse, the effects of angular errors will reduce the effects of the linear errors, or vice versa, but in general, where the laws of accidental errors are operating, the total error of closure will be comprised of both angular and linear errors and will be greater than the error produced by either class alone.

Philip Kissam,[6] M. Am. Soc. C. E.[6a]—Very definitely, the paper by Mr. Bauer is a contribution to the art of surveying. As the author states, too little is known of the actual accuracies obtained in the field in the usual property surveys, or just what can be done to obtain the desired accuracies at the minimum cost. If more surveyors would follow Mr. Bauer's lead by compiling

[5] "Specifications for Transit Traversing and Stadia Leveling," by W. H. Rayner, *Transactions*, Am. Soc. C. E., Vol. 94 (1930), p. 679.

[6] Associate Prof., Civ. Eng., Princeton Univ., Princeton, N. J.

[6a] Received by the Secretary August 4, 1942.

data as he has, there would be no question whatever that accuracies, methods, and instruments would be improved and costs reduced.

Errors in Transit Results.—To analyze Mr. Bauer's conclusions, the writer has classified the errors of the transit as follows:

Major errors in scale reading—
- (1) Errors in determining what the scale reads (probable error = r).

Minor errors in scale reading—
- (2) Errors in graduation.
- (3) Errors due to play between the inner and outer spindles.

Major errors in pointing—
- (4) Errors due to lateral refraction.
- (5) Errors due to the positions of the instrument and signals (including phase). The probable error of Items 4 and 5 is h. The two types cannot be separated using the data given.
- (6) Errors due to not pointing the cross hair on the apparent signal (probable error = p).

Minor errors in pointing—
- (7) Errors due to temperature changes in the instrument and tripod.
- (8) Errors due to lack of adjustment of the instrument.
- (9) Errors due to play in the slide and the trunnions and errors in optics.
- (10) Errors due to lack of verticality of vertical axis in setup.
- (11) Errors due to play between the outer spindle and the leveling head.

This classification depends on the assumption that the transit is in good working order and that proper operational procedure is used.

In this discussion only the major errors will be considered as the minor errors are negligible under the conditions stated.

Mr. Bauer has determined the accuracy of a single reading of a 1′ transit by tabulating 176 sets of angles. In each case the first reading and the mean of six turns were recorded, together with the difference between the first reading and the mean of six. The mean difference of the entire 176 angles tabulated was found to be 21″. According to the theory of errors the probable error would be: $0.848 \times 21'' = 17.8''$.

It will be seen that this error E_6 is made up as follows:

Referring to Items 1, 4, 5, and 6, "h" is not present as each measurement was made at one time and with the same positions for the signals and the instrument. Since each pair of observations was made at the same time it is assumed that the lateral refraction was also constant; thus:

$$E_6 = \tfrac{1}{6}\,(k\,r_1 + r_6 + p_1 + p_2 + \cdots + p_{12}) \qquad (6)$$

in which E_6 is the actual error of the mean of six repetitions, and k is the ratio between the error in setting the vernier at zero and the error of reading a 1′ transit to 30″. If E_1 is the actual error of the first reading:

$$E_1 = k\,r_1 + r_2 + p_1 + p_2 \qquad (7)$$

and

$$E_1 - E_6 = \tfrac{5}{6}\, k\, r_1 + r_2 + \tfrac{5}{6}\, p_1 + \tfrac{5}{6}\, p_2 - \tfrac{1}{6}\,(r_6 + p_3 + p_4 + \cdots + p_{12}) \quad (8)$$

The probable value of Eq. 8 is:

$$17.8 = \tfrac{1}{6}\sqrt{(25\, k^2 + 37)\, r^2 + 60\, p^2} \quad (9)$$

Mr. Bauer finds the probable error of a set of six turns to be 3.8; hence:

$$3.8 = \tfrac{1}{6}\sqrt{k^2\, r^2 + r^2 + 12\, p^2} \quad (10)$$

Solving Eqs. 9 and 10 simultaneously:

When k =	r =	and p =
0.3	16.2	4.4
0.5	15.5	4.3
0.8	14.0	4.0
1.0	13.0	3.9

Assume the third set of values as correct ($k = 0.8$).

It is possible to determine h. Mr. Bauer has determined the probable error of an angle measured by six repetitions when the measurements were made at different times. Such an error includes h. The value determined is 7.0″. Therefore, $7.0^2 = 3.8^2 + h^2$; and $h = 5.9''$. If there is no lateral refraction and the observer can assume that the error in position of the instrument is one half that of the signal, the angular error due to a setup s is $5.9 = \sqrt{2\, s^2 + 2\,(2\, s^2)}$; and $s = 1.89''$.

Since the average length of line is 700 ft, the probable error in position is expressed as follows: $d = \sin 1.89'' \times 700 = 0.0066$ ft, in which d is the error in setting up the transit.

This computation results in the following probable errors in angle determination:

r (reading scale)	= 14.0″
$0.8\, r$ (setting, zero)	= 11.0″
h (position of transit and signals at 700′)	= 5.0″
p (pointing on apparent signal)	= 4.0″
d (transit position)	= 0.0066 ft
$2\, d$ (signal position)	= 0.0132 ft

These depend on two assumptions. The first is that the vernier can be set at zero with 0.8 as the error of reading the scale. The second is that the error of the signal position is twice the error of the transit position.

There are no data in Mr. Bauer's paper from which can be deduced the decrease in the probable error that would result from reading the minute transit to 15″ or the effect of finer graduations. It is the opinion of the writer that:

1. There is a considerable reduction in probable error when the vernier of a 1′ transit is read to 15″. This procedure has been followed since 1933 by the New Jersey Geodetic Control Survey with considerable reduction in rejected angles.

2. The fineness of graduation has little effect on the accuracy, the accuracy being chiefly a function of the clarity of the graduation as presented to the observer.[7]

Accuracy of Linear Measurement.—Mr. Bauer has given the probable errors of linear measurement for immediate retaping and for retaping at a later date. As he states, this is not based on comparisons with precisely measured high-accuracy surveys. As the accuracy of taping for a given field procedure depends chiefly on tension, temperature, slope, and tape calibration, retaping by the same party on the same day with the same spring balance gives a poor determination of the accuracy, and, as Mr. Bauer points out, is of little practical value except to pick up blunders. The probable error for this condition is given as 1 : 22,800 or 0.0000439, which corresponds to an angle of 9″.

From the theory of errors (which Mr. Bauer quotes) the following is evident: Probable error $= 0.954 \frac{\text{Total error}}{2}$. Hence, the total error or accuracy for linear measurement is 1 : 10,880.

Traverse Accuracies.—It would seem that when traverses are measured the field procedure should be such that the probable error of the transit would be about equal to that of the linear measurements, as Mr. Bauer suggests. If the angles were measured once with the telescope direct and the measurement repeated once with the telescope reversed (indicated herein as 1 DR); and if both verniers were read each time, the expression for the probable error of angular measurement would be $\frac{1}{2}\sqrt{\frac{1}{4}(k^2+3)\,r^2+4\,p^2}$. Substituting the values computed, the probable error is 7.78″.

Introducing the errors of position of signals and transit, the resulting accuracy is $\sqrt{7.78^2+5.9^2} = 9.8''$. This probable error corresponds very closely with the probable error of linear measurement found by Mr. Bauer, which is the equivalent of an angular accuracy of 9.0″. Thus, where an average accuracy of 1 : 10,000 for a traverse is required, the angles should be measured 1 DR, both verniers should be read, and the usual methods of distance measurement used. Such traverses would have an accuracy of between 1 : 5,000 and 1 : 15,000.

If higher accuracies are desired, the method of taping should be improved. If it were desired to obtain traverses having an accuracy of 1 : 25,000, a probable error of 1 : 52,000 would be necessary. The use of the plumb bob would have to be eliminated. Probably this could be obtained in commercial work by using a 300-ft tape fully supported on some surface, like a sidewalk or street pavement, using a thermometer and spring balance, and running a rough profile over the tape for slope corrections. When such a procedure is used the accuracy of measurement will usually attain better than 1 : 25,000, having a probable error of 1 : 52,000.

A probable error of 1 : 52,000 corresponds to about 4″ of arc. To obtain a probable error of 4″ in the transit work would require a considerable improvement in the centering of signals and possibly in the transit itself.

[7] "Accuracy of Transit Work Depends on Clearer Circle Graduations," by Philip Kissam, *Engineering News-Record*, April 10, 1941, p. 51.

The desired results could be obtained if the signals were designed to have no phase and the work could be performed at any time when there was at least some haze or cloudiness so that the lateral refraction would be reduced, and if the signals and the transit were set so that the probable error of their positions was 0.005 ft. Under these conditions $h = \sqrt{4 s^2} = 2 s$; but, at 700 ft, $s = \frac{0.005}{700} = 1.40''$, and $h = 2.8''$.

Transit accuracy also would have to be maintained so that a lower probable error would result than Mr. Bauer found to be the rule.

To obtain this accuracy it is suggested that one set of 3 DR be used, reading both verniers at the beginning and end of the set. Then, the probable error is $\frac{1}{6} \sqrt{\frac{1}{4} (k^2 + 3) r^2 + 12 p^2} = 3.20''$. Combining the error of the transit operation and the error of the positions of the transit and the signals, the probable error of angle determination would be $\sqrt{(3.20)^2 + (2.8)^2} = 4.2''$. This is practically low enough.

This error would be reduced, of course, by using lines of more than 700 ft so that a reasonably good balance between transit accuracy and taping accuracy would be maintained. Assuming that transit errors and taping accuracy affect the traverse equally, the probable error of such a traverse would be about 1 : 50,000, and the accuracy about 1 : 24,000.

Conclusions.—1. Assuming that Mr. Bauer's traverses represent the usual traverses run by good land surveyors, the accuracy to be expected is between 1 : 5,000 and 1 : 15,000.

2. The transit work is of a much higher degree of accuracy than the taping and so long as the present method of taping persists there is no advantage in turning the angle more than 1 DR, using both verniers.

3. If high-grade traverse work is required, an accuracy of 1 : 24,000 can be obtained with little, if any, increase in cost if a 300-ft tape is used and the angles turned 3 DR reading both verniers.

These recommendations are based on the assumption that a better target is used than is ordinarily found in practice and that the noon hours on clear days be avoided for angle measurement.

It will be noted that in determining the order of accuracies of traverses, Mr. Bauer has assumed that *Manual No. 10* uses a "probable" error in specifying the order of accuracy. It is the belief of the writer that by "error of closure" is meant the total error divided by the total length, a value which is much more difficult to obtain. For example, the average position closure required for first-order traverse is 1 : 35,000. To obtain this accuracy the average probable error would have to be 1 : 73,500.

George D. Whitmore,[8] M. Am. Soc. C. E., C. C. Miner,[9] Assoc. M. Am. Soc. C. E., and W. O. Byrd,[10] Esq.[10a]—An interesting approach to the complex subject of traverse accuracies is contained in this paper. The writers have long

[8] Chf. of Surveys, Maps and Surveys Div., TVA, Chattanooga, Tenn.

[9] Civ. Engr., Maps and Surveys Div., TVA, Chattanooga, Tenn.

[10] Associate Mathematician, Maps and Surveys Div., TVA, Chattanooga, Tenn.

[10a] Received by the Secretary September 24, 1942.

felt that too much emphasis has been placed on the "just-happened" closure of the entire loop, and not enough on designing measuring instruments and measuring procedures that will insure uniform precision for each and every tangent of the entire traverse. Experience shows that traverses having circuit closures of high order, such as 1 : 10,000 or 1 : 20,000, may conceal errors, in the individual courses, of the order of 1 : 1,000. The writers believe that the accuracy specified for a traverse loop or line should be an indication also of the approximate precision of each individual course in the entire line.

Traverse does not have as many rigid geometric checks available as does triangulation, and for that reason safeguards against blunders are imperative. Within the limits of accuracy desired in the traverse, known systematic errors should be eliminated, and measuring procedures should be used which will minimize the unknown systematic and accidental errors. Measuring instruments should be used that will provide the required accuracies most economically.

Unless consideration is given to some of the systematic taping errors, third-order traverse accuracy can be attained only under exceptional circumstances. For instance, any one of the following conditions would result in systematic errors that would equal the limits for third-order traverse accuracy (1 : 5,000):

(*a*) If the actual length of a 100-ft tape differed by as much as 0.02 ft from the true length, a systematic error of 1 : 5,000 would result. Such a change in length may be caused by stretching or kinking.

(*b*) Under certain conditions a systematic error of 5 lb in the tension applied to the tape would result in a systematic error of 1 : 5,000.

(*c*) If the actual temperature of the tape is in error by 30° F, a systematic error of 1 : 5,000 would result. Such an error may result from applying no temperature correction, or, under extreme conditions, from applying the temperature of the air rather than of the tape itself.

There are, of course, other systematic taping errors, the accumulated effect of which may be appreciable.

The need for reducing systematic taping errors is felt immediately when the traverse runs between two previously fixed points, such as high-accuracy triangulation stations, or is part of a traverse net run under varying weather conditions by two or more parties using different measuring equipment and procedures. These errors, obviously, will not ordinarily be felt when dealing only with a single traverse that is run in a loop, originating and terminating on the same point, such as described by the author.

The attempt to eliminate systematic errors, both linear and angular, will influence, to a large extent, the measuring procedures and instruments used; a study of surveying accuracies is incomplete until the effects of such errors are considered. Systematic angular errors fortunately can be largely eliminated by proper observing procedures and by distributing angular closures before computing latitudes and departures.

The writers believe that accidental angular errors have a considerable effect on the quality of a traverse, but that accidental taping errors, because of their more nearly complete compensation, have a much lesser effect and account for only a small part of the total traverse closure. The effect of accidental

angular errors can be reduced, however, by means of any or all of the following safeguards: (1) Frequent azimuth control lines through triangulation; (2) frequent astronomical observations; and (3) use of long azimuth control lines, which span several of the shorter, directly taped courses.

The writers would like to add the following thoughts to the three questions posed at the end of the paper:

A. The limiting of the closure error is not adequate to assure a desired traverse accuracy. The writers believe that the specifications should include the measuring procedures and instruments to be used, so that each individual tangent will be of the required accuracy.

B. Smaller plate graduations on transits for city work are justified primarily for reasons of economy, because the desired angle accuracy can be obtained with a fewer number of turns.

C. The writers find little to criticize in the design of the newer transits now on the market. Of course, improvements will continue to be made; however, they will probably be in the direction of greater accuracies and more stability.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

MOMENT BALANCE: A SELF-CHECKING ANALYSIS OF RIGIDLY JOINTED FRAMES

Discussion

BY G. W. STOKES, ESQ.

G. W. STOKES,[13] ESQ.[13a]—One of the most useful applications of Mr. Cornish's original adaptation of the slope-deflection formulas occurs in connection with moment calculations for "ordinary building" frames.[14] No definition of such frames appears to be available but, no doubt, essential features are that the beams shall be prismatic; that the inertia moment of any haunch, considered as a rectangular section, shall not be greater than the assumed inertia moment of the midspan tee section; and that sidesway due to vertical loading is negligible.

The number of moment calculations for a complete multi-story building frame is usually large and is magnified by the alternative arrangements of superload which give maximum moments. Since the relative K-values are in the first place unknown, it is common practice to proceed by means of intelligent approximation of these values, based in the first place on consideration of fixed-end beam moments and statical reactions, afterward adjusted by the simplest possible calculations of end moments for limited sections of the frame. For these preliminary estimates of moments it is doubtful whether any simpler method has been found, or is likely to be found, than single-cycle distribution and carry-over of individual fixed-end moments for dead load, summation of such moments and further summation by the addition of moments for alternative superload arrangements, obtained by proportion from the relation of superload to dead load.

It is in the final stage of calculation, which follows the preliminary estimates, that the characteristics of the "moment-balance" method appear to be most

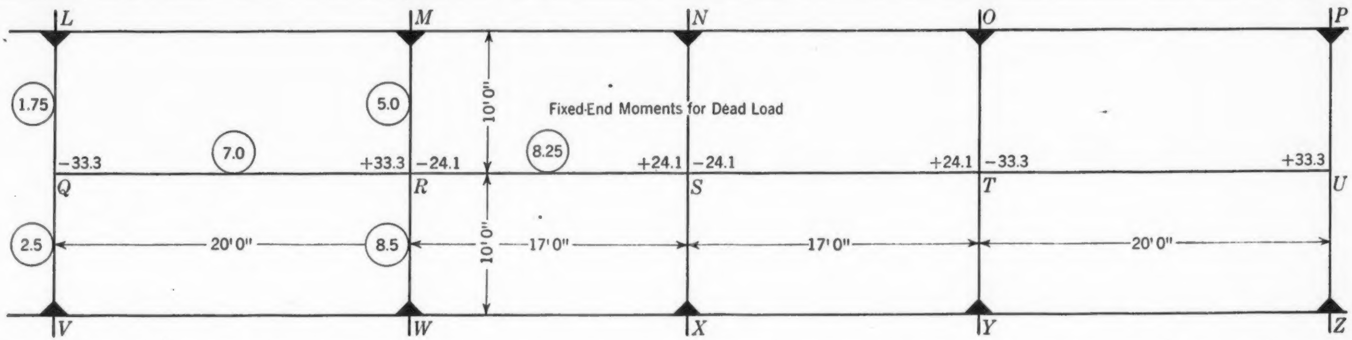
NOTE.—This paper by R. J. Cornish, Esq., was published in May, 1942, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1942, by Messrs, D. D. Matthews, and William A. Larsen; June, 1942, by Messrs. A. A. Eremin, L. E. Grinter, and B. J. Aleck; and September, 1942, by Messrs. Frederick S. Merritt, and Ralph W. Stewart.

[13] Cheadle Hulme, Cheshire, England.

[13a] Received by the Secretary August 29, 1942.

[14] "Recommended Practice and Standard Specifications for Concrete and Reinforced Concrete," Rept. of the Joint Committee on Standard Specifications for Concrete and Reinforced Concrete, *Proceedings*, Am. Soc. C. E., June, 1940, Pt. 2, paragraph 805(*c*), p. 46.

TABLE 5.—Solution of a Simple Building Frame



Quantity	Joint Q				Joint R					Joint S					Joint T					Joint U			
	Member QL	Member QR	Member QV	Σ	Member RQ	Member RM	Member RS	Member RW	Σ	Member SR	Member SN	Member ST	Member SX	Σ	Member TS	Member TO	Member TU	Member TY	Σ	Member UT	Member UP	Member UZ	Σ
	(a) Span Q-R Loaded																						
K	1.75	7.0	2.5	11.25	7.0	5.0	8.25	8.5	28.75	8.25	5.0	8.25	8.5	30.0	8.25	5.0	7.0	8.5	28.75	K = stiffness ratio I/L			
M_F	....	− 3.33	....	....	+ 3.33	....	....	....	....	....	....	....	....	....	....	....	....	....	....	M_F = fixed-end moment			
$-X$	0	−100.0	0	−100.0	+100.0	0	0	0	+100.0	0	0	0	0	0	0	0	0	0	0	X = a slope-deflection term			
M'	+ 3.0	+ 34.0	+ 4.2	+ 41.2	− 14.8	− 3.9	− 4.7	− 6.6	− 30.0	−12.4	+0.5	0.6	+ 0.9	−10.4	+1.7	−0.1	0	−0.1	+1.5	M' = external moment at outer end			
κ	+ 9.2	+ 36.5	+13.1	+ 58.8	− 17.0	−12.2	−20.1	−20.7	− 70.0	+ 2.9	+1.7	+ 2.8	+ 3.0	+10.4	−0.5	−0.1	−0.4	−0.5	−1.5	κ = a substitution factor			
$2M$	+12.2	− 29.5	+17.3	....	+ 68.2	−16.1	−24.8	−27.3	....	− 9.5	+2.2	+ 3.4	+ 3.9	....	+1.2	−0.2	−0.4	−0.6	....	M = final moment			
M	+ 6.1	− 14.8	+ 8.7	....	+ 34.1	− 8.0	−12.4	−13.7	....	− 4.8	+1.1	+ 1.7	+ 2.0	....	+0.6	−0.1	−0.2	−0.3	....				
	(b) Span R-S Loaded																						
K	1.75	7.0	2.5	11.25	7.0	5.0	8.25	8.5	28.75	8.25	5.0	8.25	8.5	30.0	8.25	5.0	7.0	8.5	28.75	7.0	1.75	2.5	11.25
M_F	....	....	....	....	....	....	−24.1	....	....	+24.1	....	....	....	....	....	....	....	....	....	....	....	....	....
$-X$	0	0	0	0	0	0	−72.3	0	− 72.3	+72.3	0	0	0	+72.3	0	0	0	0	0	0	0	0	0
M'	− 0.2	+ 6.1	− 0.5	+ 5.4	+ 1.3	+ 2.4	+20.0	+ 4.0	+ 27.7	−19.8	−2.3	− 2.7	− 4.0	−28.8	−7.4	+0.1	+0.2	+0.1	−7.0	+0.9	−0.1	−0.1	+0.7
κ	− 0.8	− 3.4	− 1.2	− 5.4	+ 10.8	+ 7.8	+12.8	+13.2	+ 44.6	−12.0	−7.2	−12.0	−12.3	−43.5	+2.0	+1.2	+1.7	+2.1	+7.0	−0.4	−0.1	−0.2	−0.7
$2M$	− 1.0	+ 2.7	− 1.7	....	+ 12.1	+10.2	−39.5	+17.2	....	+40.5	−9.5	−14.7	−16.3	....	−5.4	+1.3	+1.9	+2.2	....	+0.5	−0.2	−0.3	....
M	− 0.5	+ 1.4	− 0.9	....	+ 6.1	+ 5.1	−19.8	+ 8.6	....	+20.3	−4.6	− 7.4	− 8.1	....	−2.7	+0.7	+0.9	+1.1	....	+0.3	−0.1	−0.2	....

valuable—namely, rapid convergence, simple tabulation and ease in checking. The method is somewhat lengthy if several balances are required. It is important, therefore, that the estimates of the M'-values should be reasonably close.

By the insertion, in their appropriate columns, of the M'-values and K-values already obtained, one "balancing" operation is usually sufficient to give results accurate enough to form the basis for design.

A point in favor of Mr. Cornish's method, for many designers, will be its clear derivation from the well-known slope-deflection formulas. The substitution of values in formulas is a practice well established among structural engineers, a fact which no doubt explains the many efforts to express the processes of moment distribution in terms of symbols.[15]

An example of a section of a simple building frame is shown in Table 5. The M'-values and K-values are those obtained by previous approximation. Check calculations made by the usual slope-deflection and full moment-distribution methods give results practically identical with the M'-values shown in Table 5. The necessary summations are common to any method and are not given.

[15] For example, see "Recommended Practice and Standard Specifications for Concrete and Reinforced Concrete," Rept. of the Joint Committee on Standard Specifications for Concrete and Reinforced Concrete, *Proceedings*, Am. Soc. C. E., June, 1940, Pt. 2, Appendix 2, p. 110.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

CLASSIFICATION OF IRRIGABLE LANDS

Discussion

By Messrs. Walter W. Weir, and W. L. Powers

Walter W. Weir,[3] Assoc. M. Am. Soc. C. E.[3a]—It was a step in the right direction when the state and federal agencies concerned with the development of new agricultural lands learned that, when the engineering problems of supplying water had been solved, there was still the question of the suitability of the land to be taken into consideration. The sequence of these two actions is of vital importance and when properly timed may avoid many disappointments.

It is common knowledge that water has been made available to some lands that could not be profitably farmed even though water were free. Although it is not a recent discovery of the Bureau of Reclamation that the inherent quality of the soil is of major importance, the Bureau is not entirely without fault even in some of its later projects. As long ago as 1925 the writer was employed to make a soil classification of a Reclamation Service project as an aid in making adjustments in water charges which had been found to be out of line with the producing power of the soil.

Considerable advancement has been made in the land-classification concept since the passage of the "Fact-Finders Act" in 1924, and of course that is as it should be. Mr. Johnston has presented the results of some 18 years of Bureau of Reclamation experience in attempting to arrive at a sound basis for land classification.

The author intimates that the purpose of land classifications is to determine what lands are arable (using his definition of the word "arable"), and that this information will be used in the design of the irrigation system. Unfortunately, the timing of these land-classification surveys is faulty, and even some of the recent surveys have not been completed until after construction is well under way or completed.

For many years the University of California, in Berkeley, has urged a detailed soil survey and land classification of the East Imperial Mesa in California, an area of some 225,000 acres of desert, for which water might be

Note.—This paper by W. W. Johnston, Esq., was published in May, 1942, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1942, by R. Earl Storie, Esq.

[3] Drainage Engr., Div. of Soils, Univ. of California, Berkeley, Calif.

[3a] Received by the Secretary August 31, 1942.

made available from the All-American Canal. The classification of the soils on this mesa is an example of poor timing, as it was not until after the completion of the All-American Canal and the Coachella Branch (from which branch much of the land is to be irrigated) that the Bureau of Reclamation undertook a land-classification survey. The writer is a member of the Advisory Committee on Land Classification for this area and assisted in setting up the standards of classification. These followed substantially the system described in Mr. Johnston's paper.

The Coachella Branch was constructed with capacity sufficient to irrigate nearly 200,000 acres on this mesa. This was based on a very rough reconnaissance soil survey made in 1920, which would not now qualify as a reconnaissance land classification as described by Mr. Johnston. The detailed classification completed in 1942 places the arable lands on the mesa at a substantially lower amount—in fact, so much lower that, had the information been available sooner, the construction of a canal of substantially different cross section might have resulted. Although the writer does not have the data at hand, it is probable that a somewhat similar situation may be found in the Columbia Basin area in the State of Washington.

With regard to the East Imperial Mesa survey, the Advisory Committee was in full agreement on the standards established, having in mind the particular use that was to be made of the survey, but the Committee strongly advised that a detailed soil survey be made either before or concurrently with the land classification. This suggestion has been followed; the University of California cooperated with the Bureau of Reclamation and the two surveys were made concurrently, to the advantage of both agencies.

One criticism that may be made of the land classifications as outlined in this paper is that the procedure is such that it can be followed by engineers whose training in soils may be limited. The Bureau of Reclamation personnel consists largely of men of engineering training and it has been exceedingly slow in going far afield for men trained in other branches of science. Land classification is not strictly an engineering job and, aside from the reconnaissance type of survey, should not be made by engineers.

The writer frequently has disagreed with the Bureau of Reclamation in its contention that detailed soil survey data are not directly applicable to its needs and that the Storie Index for soil rating cannot be applied generally to soils outside of California because the basic principles used in the index are profile development and texture of the surface soil.

Closer correlation in land classification can be maintained among different areas in the same or different states through the use of the Storie Index and R. Earl Storie's "Natural Land Types" than by any method with which the writer is familiar; but they must be applied by men familiar with soil. Land classification, made for whatever purpose, is only a special adaptation of soil survey information.

Mr. Johnston's statement that soil characteristics vary so greatly that it is difficult to make a uniform application as a guide to irrigability, and his comparison of conditions in the Sacramento Valley and eastern Montana, will hardly support a contention that detailed soil surveys are not the most logical

basis for land classification. This contention may have some merit if the classifiers are not soil trained, but it certainly is not true if a little closer coordination could be secured between the engineer and the soil technician.

It has been found in the work of Mr. Storie and the writer at the University of California that, the more detailed the soil survey, the more accurately a land classification can be made. Their most severe criticism has come from attempts to make classifications in the absence of up-to-date soil data. By up-to-date soil data is meant a detailed soil survey which not only delineates and describes the soil profile in terms of its development and texture of the various horizons and strata, but also contains information on alkali, drainage, water-holding capacity, slope, and erosion. Many of the published soil survey reports are admittedly not satisfactory for the purpose, and only the most recent surveys are adequate. This condition is only to be expected, however, as soil surveying as well as land classification has developed rather rapidly, from a modest beginning, into a science.

The writer can agree with Mr. Johnston in his statement that future drainage requirements on new irrigation projects are difficult and often impossible to determine in advance. He has never fully agreed with some persons (not engineers) who have advocated the construction of drainage works at the same time as the construction of irrigation works. On the other hand, a knowledge of the soil, its age, or profile development, mode of formation, parent material, and other features that may be furnished by a soil survey are a material aid in predicting what lands may be subject to seepage, waterlogging, or salt accumulation, and their placement in the proper land-classification group.

W. L. Powers,[4] Esq.[4a]—The land-classification work in which Mr. Johnston and the writer were so closely associated before and after the enactment of the 1924 amendment to the Federal Reclamation Act has been developed to a high state of precision and usefulness. Members of the five-man board of review of class 3s and class 3d lands had an opportunity to test the accuracy and value of the Columbia Basin project classification in July, 1941. A similar procedure may be used in a soil survey and classification of irrigation-project lands to be undertaken for the Venezuelan Department of Public Works beginning approximately September 15, 1942.

The usable water capacity of the soil is perhaps its most important physical property in land-classification work for irrigation. The water capacity of the soil profile used by plant roots seems to be a more useful criterion than definite limitations as to soil depth. In successful projects with surface irrigation in Northwestern United States, this should be at least 4 acre-in. Some improvement in soil profile water capacity may be effected by the use of deep-rooted, humus-building legumes and farm manures from livestock enterprises.

If irrigation is to be permanent and is to repay those financially concerned, it should be preceded by a rigid land classification. This paper agrees closely with the writer's experience.

---

[4] Soil Scientist in Chg., Oregon Agri. College and Experiment Station, Oregon State College Corvallis, Ore.

[4a] Received by the Secretary September 8, 1942.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

NUMERICAL PROCEDURE FOR COMPUTING DEFLECTIONS, MOMENTS, AND BUCKLING LOADS

Discussion

BY ALFRED S. NILES, ASSOC. M. AM. SOC. C. E.

ALFRED S. NILES,[25] ASSOC. M. AM. SOC. C. E.[25a]—The method of computation described in this paper is very ingenious, and should prove to be a great timesaver in the solution of many types of problems. Although the author has shown applications to both beams and columns of single span, he has failed to warn the reader that his method is not directly applicable to continuous members, for which the bending moments over the supports must be obtained by the use of the three-moment equation, the method of moment distribution, or an equivalent method. Perhaps the most awkward member of this type, from the point of view of the stress analyst, is a continuous beam of nonuniform section that is subjected to combined bending and compression. This is handled most readily by the method of moment distribution, proper allowance being made for the effect of the axial load when computing fixed-end moments and the carry-over and stiffness factors. The computation of these quantities for members of nonuniform section by previously published methods is a slow and tedious procedure. The methods proposed by the author appear more convenient for this work than any other that has yet been suggested, including that of the writer and J. S. Newell.[7]

In his numerical examples the author divides the beam into segments of equal length. This greatly simplifies the work, and is nearly always allowable when the transverse load (or "angle change") can be completely represented by a smooth curve. If this is not allowable, however, as when unequally spaced concentrated loads are present, much of the advantage is lost. In fact, the author's method becomes practically the same as that described,[26] the only

NOTE.—This paper by N. M. Newmark, Assoc. M. Am. Soc. C. E., was published in May, 1942, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1942, by Bruce Johnston, Assoc. M. Am. Soc. C. E.; June, 1942, by M. S. Ketchum, Jr., Assoc. M. Am. Soc. C. E.; and September, 1942, by Messrs. John B. Wilbur, Ralph W. Stewart, and Stefan J. Fraenkel.

[25] Prof., Aeronautic Eng., Leland Stanford Junior Univ., Stanford Univ., Stanford University, Calif.

[25a] Received by the Secretary August 17, 1942.

[7] "Airplane Structures," by A. S. Niles and J. S. Newell, 2d Ed., New York, N. Y., 1938, Vol. I, pp. 133–136, and Vol. II, pp. 126–134.

[26] *Ibid.*, Vol. I, pp. 58–60.

differences being in the method of recording the computations, and in the more accurate, although more time-consuming, method of allowing for the curvature of the loading (or "angle change") diagram, which the writer treats as composed of straight segments. In most practical problems, however, the value of the greater accuracy of the author's method of allowing for this factor is questionable.

In computing "equivalent concentrated angle changes," the author uses formulas from Figs. 3 and 5 that require division by 6, 12, or 24, depending on the shape of the angle-change curve. It would seem simpler to include the factor 6, 12, or 24 in the "common factor" by which the moments or deflections are to be multiplied to get the final results. This would result in most of the values in the tabulated computations being 6, 12, or 24 times as large as those obtained in the method as described, but would not affect the final results. Care would have to be taken, however, to use the same factor throughout the span when part of the loading curve was straight and part curved, but in any given problem it would be easy to multiply the formulas of Figs. 3 and 5 by $\frac{2}{2}$ or $\frac{4}{4}$ in order to obtain a common denominator for all the formulas used.

In some of his examples (as in Fig. 1(*d*)) the author computes the actual value of the shear at the left end of the beam before computing the shears at other points, whereas in others (as in Fig. 1(*e*)) he starts the shear computations from an arbitrarily assumed figure and makes a final correction to the bending moments, if necessary. The writer has two objections to the latter practice, although it seems to be preferred by the author. The first is that it is often necessary to know the shears at various points along the span, and, in the latter practice, it would be necessary to remember to correct the values originally found to obtain the true ones. This could be done easily, and the objection would be unimportant if it stood alone. The more serious objection is that the practice eliminates a valuable internal check on the computations. If the actual shear at the left end is first computed, then the moment at the right end, computed by summation of the shears along the span, should be the same as that stated in the formulation of the problem. If the two values are not in substantial agreement, an error has been made. In the method of Fig. 1(*e*), one does not know whether the necessary moment correction is due solely to the difference between the assumed and actual shears at the left end, or whether it is partly due to a numerical error of computation. Since the actual shear at the left end can be computed quite easily, the check obtained justifies the little additional work involved in using it.

The writer notes that the author has reversed the usual convention and considers that loads are positive when they act upward. He heartily indorses this practice. He wonders, however, why the same change was not made in the conventions for slope and deflection. That would have involved the elimination of the minus sign from the definition of "angle change" as $-\frac{M}{E\,I}$; but that sign is not essential. It is there only to reconcile some independently assumed conventions which proved to be lacking in logical consistency. It is really much simpler to assume upward loads and deflections as positive. Then

one can differentiate the equation of the elastic curve four times, obtaining successively the slope, bending moment, shear, and loading, without having to remember to reverse signs arbitrarily at various steps.

A related point is in connection with nomenclature. In his paper the author terms the quantity, $-\frac{M}{E\,I}$, the "angle change." Actually, as he states, he is using that expression as an allowable approximation for what mathematicians call the "curvature," $\frac{1}{r}$, in which r is the radius of curvature. Since the mathematicians already have given the quantity a name, why was it necessary to rechristen it? It might be considered awkward to speak of a "curvature curve," although the expression should be quite as clear as "angle-change curve"; and "concentrated curvature" should be as clear as "concentrated angle change." What the author has termed "angle change" is really "rate of slope change," and the latter term would really be preferable to the former, if "curvature" is to be replaced by something else.

In studying the numerical examples, the writer was unable to verify one of the author's figures. In Fig. 15(a) the equivalent concentrated angle change at the section of change in moment of inertia is shown as -404.90. This appears to be a quantity to be obtained by use of the formulas of Fig. 5(a), assuming the distributed angle-change curves produced to have ordinates either one tenth of, or ten times, those of the actual curve, in the adjacent segments of the beam. On this basis, the concentrated angle change in question would appear to be $-(3 \times 513.5 + 10 \times 803.6 - 1 \times 911.0 + 3 \times 91.10 + 10 \times 80.36 - 1 \times 51.35) \times \frac{1}{24} = -403.79$. The actual difference between this value and the author's is of no practical consequence, but it would be interesting to learn whether the figure in the text was computed by some other method.

Although the author's paper is subject to the foregoing minor criticisms, he deserves much credit for developing a valuable new tool for the use of the structural engineer.

Correction for *Transactions:* In May, 1942, *Proceedings*, page 715, Fig. 20(b), line 5, "Assumed Average Slope," change "-148" to "-448."

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

STATISTICAL ANALYSIS IN HYDROLOGY

Discussion

BY MESSRS. L. STANDISH HALL, AND H. ALDEN FOSTER

L. STANDISH HALL,[7] M. AM. SOC. C. E.[7a]—In presenting applications of statistical analysis to hydrology, this paper should prove valuable in many respects. The accuracy of the projection of a short-term record by statistical methods to determine the probability of the occurrence of an event over a longer period of time has long been open to question. In fact, even on the basis of statistics, the probable error of such a projection is large.[8]

The theory of the frequency distribution curve, on which the duration curve is based, is predicated on certain assumptions indicated by the author. These conditions are not rigidly applicable to hydrological data, since it is tacitly assumed:

(1) That there is no difference in any essential respect between the conditions and methods from which the observations are drawn; and, if the observations have been made at different epochs, that there has not been any essential change during that period;

(2) That the conditions that regulate the appearance of any event must be the same for every series, and also for every observation in the series; and

(3) That the individual events must be completely independent of one another.

Of these conditions, the second and third are marked limitations. To satisfy the first condition in stream-flow measurements, no changes should have occurred from the transfer of the measurements from one location to another, or in the accuracy with which gage heights are observed or stream flow measured or estimated. It is well known that this condition has not been fulfilled, since there has been a progressive improvement over the period of available records with regard to all of these items. Recording gages have replaced staff gage readings; improvements have been made in the technique of measuring stream flow and of estimating flood discharges by surface measure-

NOTE.—This paper by L. R. Beard, Jun. Am. Soc. C. E., was published in September, 1942, *Proceedings*.

[7] Hydr. Engr., East Bay Municipal Utility Dist., Oakland, Calif.

[7a] Received by the Secretary August 29, 1942.

[8] *Transactions*, Am. Soc. C. E., Vol. 91 (1927), p. 58.

ments, slope observations, or water-surface drop at contracted openings.[9] In general, no attempt has been made to review previous flood discharge observations in the light of more recent knowledge. The same situation also applies to rainfall observations and most long records are a composite of observations at several locations in a given locality made under varying conditions of accuracy, due to exposure to wind and height of gage above ground. Also, assuming that the measurements of stream flow were all of equal accuracy, there have been changes on many streams due to the works of man causing changes in channel storage by reservoirs, levees, and channel improvement, changes of permeability of the surface by roads, buildings, cultivation of land, soil erosion, and deforestation by lumbering or by fire.

The satisfaction of the second condition requires that no cyclic changes affect the hydrologic record. Although the climatic divisions of the world appear, in general, to be constant, there are known minor fluctuations having a duration ranging from a few years to many centuries. Of the shorter cycles the best known are the 11-yr sunspot cycle and the 35-yr Brückner cycle. Of the longer meteorological periodicities, the glacial and inter-glacial cycles are the most pronounced; but even within historic times there is definite evidence of wet and dry cycles. In the western United States evidence of climatic changes have been derived from the width of the annual growth of the Big Trees of California, checked and adjusted by the variations in the levels in salt lakes of the desert area and by archeological remains. As a result of the correlation of these data, the cycles of precipitation have been traced back to 1,000 B.C., indicating the occurrence of alternate wet and dry periods, each having durations of several hundreds of years.[10] Deviations from the mean extending over cycles of such length could not be ascribed to fortuitous chance.

The third condition that individual events be completely independent of each other cannot be realized in the hydrologic cycle. Since climate is a continuous process, each event is influenced to a greater or less degree by the weather of the preceding days, weeks, months, and in some instances even years. In particular, flood discharges are circumscribed in this respect. A given precipitation may result in either a moderate runoff or a flood of unprecedented magnitude, depending on the temperature and precipitation of the preceding weeks and months, as these may affect the soil infiltration rate, or the accumulation of snow on the ground of a sufficiently shallow depth to permit of melting during a warm rain. This complex set of circumstances surrounding the occurrence of major floods results in a marked skewness of the data, which in turn affect unfavorably the accuracy of prediction from short records of future flood frequency. In some flood records of 30 to 50 years length, there has been included a maximum so greatly in excess of the frequency array that its occurrence could not possibly be predicted by mathematical analysis. More accurate flood-prediction frequencies would be possible if more study were made of precipitation intensities in conjunction with a correlation with the other factors affecting runoff. In general, rainfall records do not

[9] "The Measurement and Computation of Flood Discharge," by Carl G. Paulsen, *Transactions*, Am. Geophysical Union, 1939, Pt. II, p. 177.

[10] "Climatic History," *Encyclopædia Britannica*, 14th Ed., Vol. 5, p. 827.

exhibit the degree of skewness evidenced by flood-flow records, and hence can be projected with relatively greater accuracy. For example, in a tropical or semi-tropical climate lacking the influence of snow accumulation, the runoff from identical storms can vary considerably, depending on whether the storm occurs on soil either dry, of average moisture content, or saturated. The last condition will naturally produce the maximum runoff, and assuming that this frequency is one fourth, a precipitation occurring once in twenty-five years would produce a maximum flood only once in one hundred years. Such full use of available data, although involving much statistical analysis, would undoubtedly yield results of greater dependability. In regions having accumulations of snow on the ground, another variable is added involving such factors as saturation of the soil before the snow fell, water content of the snow, depth of accumulation, temperature, and precipitation. A favorable coincidence of these factors produces a flood of unprecedented magnitude.

The author's determination of the location of the plotting position of each of a series of occurrences is based on the theoretical expression of the binomial expansion as the form of the frequency distribution.[11] The lack of a complete mathematical treatment makes this portion of the paper obscure, and it would be difficult for one to compute the plotting position of any number of occurrences without the author's formula. This is unfortunate, as the author has not included a complete table of plotting positions.

The basis of the formula is the proposition that the frequencies of 0, 1, 2, $\cdots$ n successes of n events are given by the successive terms of the binomial expansion $(q + p)^n$; namely:

$$q^n + n\,q^{n-1}\,p + \frac{n\,(n-1)}{1 \times 2}\,q^{n-2}\,p^2 + \frac{n\,(n-1)\,(n-2)}{1 \times 2 \times 3}\,q^{n-3}\,p^3 \ldots \quad (4)$$

in which p represents the frequency of successes and q the frequency of failures. The plotting position is determined by equating the summation of successive terms of the binomial series to $\frac{1}{2}$.

There is some question in the writer's mind as to the propriety of this assumption. Considering, for example, a 10-yr period, the first term does not indicate the probability of equal chances of failure or success to secure the median of the events occurring one tenth of the time, but rather the equal chances of failure weighted against the success of obtaining 1, 2, 3, or 4 successes in the series.

This is illustrated in the following tabulation, which shows the calculated distribution for $n = 20$, $p = 0.1$, and $q = 0.9$, or the expansion of $(0.9 + 0.1)^{20}$:

No. of successes	Probability of p per 100 series	No. of successes	Probability of p per 100 series
0	12.16	5	3.18
1	27.02	6	0.89
2	28.51	7	0.20
3	19.01	8	0.04
4	8.98	9	0.01

[11] "An Introduction to the Theory of Statistics," by G. Udny Yule, Charles Griffin & Co., Ltd., 1917, pp. 291–295.

Two successes would be normal and would have the greatest probability, but cases of one success are almost as frequent and even nine successes may occur about once in 10,000 trials. The probability of obtaining 2 or more successes in 100 series of 20 events each is 60.82 as against 39.18 chances of no successes or only one success.

In all cases in which p does not equal q, an increase in n naturally raises the number of successes most probable of occurrence, but at the same time it increases the dispersion and decreases the skewness. This is revealed if the distribution for $n = 20$ is compared with that for $n = 100$ or the expansion of the series $(0.9 + 0.1)^{100}$; namely:

No. of successes	Probability of p per 100 series	No. of successes	Probability of p per 100 series
0	...	12	9.88
1	0.03	13	7.43
2	0.16	14	5.13
3	0.59	15	3.27
4	1.59	16	1.93
5	3.39	17	1.06
6	5.96	18	0.54
7	8.89	19	0.26
8	11.48	20	0.12
9	13.04	21	0.05
10	13.19	22	0.02
11	11.98	23	0.01

In this case, ten successes would be normal and would have the greatest probability, but the frequency of from 8 to 11 successes is nearly equal. The skewness of the series is much less pronounced than in the previous example. The probability of obtaining 10 or more successes in 100 series of 100 events each is 54.87 as against 45.13 chances of obtaining from zero to nine successes. It will be noted that practically no chance of failure exists in a 100-yr record. To have a probability of 1 in 100 of no success for $p = 0.1$, a 44-yr record is necessary. This indicates the length of record necessary for a similar probability for $p = 0.01$.

The chances of no successes in the series $(q + p)^n$ where $n = 1/p$ increases as p becomes smaller as follows:

p	q	n	Probability of no successes per 100 trials
0.10	0.90	10	34.87
0.05	0.95	20	35.85
0.02	0.98	50	36.42
0.01	0.99	100	36.60

In other words, the chances of failure to obtain during the once-in-a-hundred event in a series of 100 trials are greater than the chances of failure to obtain during the once-in-ten event in a series of 10 trials. This is offset by the probability of one or more successes in each series of trials. Hence, in any

given series, it is justified to assume that each occurrence is representative of the period and can be considered the median for $1/n$th of the period.

In selecting the data to form the duration curve for maximum daily precipitation or maximum flood discharges, the maximum for each of the n years can be chosen or n maximum events can be selected regardless of the year in which they occurred. Assuming that the "store" of occurrences or "universe" were known, then, with records of 10 to 50 years' length according to the binomial expansion, the chance of failure would occur in about 35% of the years and the successes would be grouped in the remaining 65% of the years. The selection of the n maximum events would be the proper manner of arranging the data. In records of this type it is usual to find wet years with two, or even three, occurrences that exceed the maximums of the dry years. In fact, on many western streams it is not unusual to find wet years having 50% or more of the daily discharges in excess of the maximum daily flow of the driest years. Under these circumstances the selection of the maximum occurrence of each calendar year obviously does not afford the best use of the available data.

The author indicates that the Gauss-Laplace normal distribution curve is the only one having a rigid mathematical derivation. Many rigid mathematical treatments are possible based on assumptions differing from those forming the basis of the normal curve of error. It is possible that many distributions covering all types may follow this law, but, in general, hydrologic data differ too greatly from it to make possible any forced agreement.

The author states (see heading "The Normal Duration Curve") that the "logarithmic transformation of the variate often has been used with exceptional success." This assumes that the geometric mean is the true average of the series, and that the logarithms follow the normal law of error. The geometric mean of a series is always less than the arithmetic mean, and most hydrologic data have a distribution with the mode less than the mean. This does not indicate that the geometric mean is the most probable value. In many series having a large number of zeros the geometric mean obviously does not apply, as for example in the distribution of daily precipitation. A very cursory examination of such data will indicate zero as the most probable value. Such data cannot possibly follow the normal law of error.

The method used in plotting the duration curves in Fig. 4 is not stated, and if the author's method has been used the location of the plotting positions would be facilitated by appending a table. The selection of the data (which is 24-hr rainfall, in inches) is perhaps unfortunate, due to the fact that in many wet years there might be two or more occurrences of 24-hr precipitation in excess of the maximum occurrences in dry years. The method used in preparing the statistical array is not stated. A discontinuous type of occurrence, as the ones illustrated, is best treated by other methods.[12] For flood discharges, for example, a combination of probability methods to determine rainfall, combined with "rational" methods for flood runoff or a unit hydrograph is most advisable. However, this criticism does not affect the validity of the author's method of analysis.

[12] "Methods of Estimating Floods" from "Floods in the United States," *Water Supply Paper No. 771*, U. S. Geological Survey, pp. 28–67.

The discussion of the accuracy of the duration curve is very valuable, but the author does not explain his method of determining the probable error and the largest deviation to sufficient extent to indicate the applicability of his results to other statistics. Much of the author's paper is argumentative rather than analytical, but this is not without value, since it presents the need for a complete analysis of statistical methods as applied to hydrology in order to evaluate the worth of various probability methods brought forth over the past 25 or 30 years and to determine the proper application of the various types of probability studies to engineering statistics. The investigation of this field of hydrology would merit further study.

H. Alden Foster,[13] M. Am. Soc. C. E.[13a]—A discussion of various questions connected with the use of duration curves in the analysis of hydrological records is contained in this paper. A number of these questions have been treated in some detail by the writer in previous publications.[14, 15, 16] At the risk of some repetition, the following comments are submitted:

Mr. Beard devotes considerable space to a demonstration of his conclusion that the generally accepted method for plotting items of the duration curve by Eq. 3 is in error, although he concedes that, in general, the presumed error is negligible. The writer has been unable to follow the demonstration of Mr. Beard's theorem.

In the first place, he states (see heading "Construction of the Duration Curve") that "the largest occurrence in a 10-yr period is considered to be the median of the largest occurrences of all of the 10-yr periods ever to happen." In the writer's opinion, this statement is not correct. As the duration curve is generally considered, the largest occurrence in a 10-yr period should be taken as the median of the 10 largest occurrences in a 100-yr record, or the median of the 100 largest occurrences in a 1,000-yr record.

To make this clear, assume 10 duration series, each covering 10 years, taken from a 100-yr record:

$$\begin{array}{l} a_1\, b_1\, c_1 \cdots\cdots\cdots h_1\, i_1\, j_1 \\ a_2\, b_2\, c_2 \cdots\cdots\cdots h_2\, i_2\, j_2 \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ a_{10}\, b_{10}\, c_{10} \cdots\cdots h_{10}\, i_{10}\, j_{10} \end{array}$$

Also assume that these 10 series have been arranged so that the a terms are in descending order of magnitude from a_1 to a_{10}. By Mr. Beard's method, if only one 10-yr series is available, the a term in that series is assumed as equal to the median of the 10 values, a_1 to a_{10}.

Now it is quite possible that one or more of the ten b terms may be greater than the smallest a term, a_{10}. Consequently, if a duration series is prepared

[13] With Parsons, Klapp, Brinckerhoff & Douglas, New York, N. Y.

[13a] Received by the Secretary August 29, 1942.

[14] "Theoretical Frequency Curves and Their Application to Engineering Problems," by H. Alden Foster, *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 142.

[15] "Duration Curves," by H. Alden Foster, *ibid.*, Vol. 99 (1934), p. 1213.

[16] "Floods in the United States," *Water Supply Paper No. 771*, U. S. Geological Survey (Chapter on Methods for Estimating Floods, by H. Alden Foster).

using the entire 100-yr record, the first 10 terms in this combined series may include some of the b terms, in which case it will not include all of the a terms. Hence, the median of the "largest occurrences of all of the 10-yr periods" (a_1 to a_{10}) will be smaller than the median of the 10 largest occurrences in the 100-yr period.

It is true (see heading "Construction of the Duration Curve") that "the probability that the maximum occurrence of a set chosen at random will exceed the median of the maximum occurrences of all the sets is 50%." The probability that the said maximum occurrence will be less than the median is also 50%. However, the probability that "none of the ten occurrences chosen at random will exceed the median of the maximum occurrences of all the sets" is not 50% as stated by the author. The 50% probability previously noted applies to the 10 maximum occurrences (the aforementioned a terms); so that the probability that any one of the ten terms in one series (such as a_1 to j_1, for example) will exceed the median of the ten a terms is $0.50 \times 0.10 = 0.05$, or 5%. Hence the probability that none of the ten occurrences will exceed the median should be 95% instead of 50%.

It seems obvious to the writer, therefore, that Mr. Beard's method of computing the percentage of occurrence corresponding to the median of the maximum occurrences from all of the sets cannot be justified. The same conclusion applies to his method for computing the other plotting positions. In the writer's opinion, Eq. 3 gives the theoretically correct plotting position. Incidentally, this method has been quite generally accepted for the plotting of duration curves in hydrological studies during the 25 years since the first World War.

The writer is in full agreement with most of Mr. Beard's discussion of duration curves, except as to the general use of the normal, or Gaussian, probability curve. It has been quite well established, both in actuarial studies and in various statistical studies in other sciences, such as biology, that unsymmetrical distributions of many types do occur. There seems to be no reason why similar distributions should not occur in hydrology and meteorology. In fact, it is the writer's observation that the unsymmetrical, or skew, distribution is the rule rather than the exception, in hydrological data. The mathematical form of the skew probability or duration curve has been discussed extensively elsewhere,[14, 15, 16] and as noted by Mr. Beard, and would seem to be outside the scope of the present discussion.

Mr. Beard's study of the accuracy of the duration curve is of interest. The writer fully concurs in his statement (see heading "Accuracy of the Duration Curve") that "one of the advantages of design by the duration curve rather than by the maximum occurrence of record is that greater accuracy is obtained." The discussion of the use of such terms as "maximum probable occurrence" is also well taken.

In conclusion, Mr. Beard should be commended for his description of the duration curve and its application to problems in hydrology. Although the writer does not agree with Mr. Beard's method of locating plotting points for the curve, this discrepancy is of only minor significance in the practical application of the curve.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

ALLOCATION OF THE TENNESSEE VALLEY AUTHORITY PROJECTS

Discussion

BY C. W. WATCHORN, ESQ., AND F. A. ALLNER, M. AM. SOC. C. E.

C. W. WATCHORN,[12] ESQ., AND F. A. ALLNER,[13] M. AM. SOC. C. E.[13a]—The problem of allocating the cost of an enterprise to the various services it renders or to the commodities it produces is difficult, even when the services rendered or the commodities produced are all of the same general kind. The problem of allocation is still more complicated for multiple-purpose enterprises or projects of which the services or products are of very different kinds.

Strictly speaking, the so-called theories of joint-cost allocations, with the possible exception of the use-of-facilities theory, are never actually what their names indicate; rather, they are mathematical methods that arbitrarily assign responsibility for the joint cost to the various purposes of a project. These cost-allocating methods are just as applicable to a single-purpose project with respect to the various users of its services as to a multiple-purpose project with respect to its various purposes. It is not difficult to imagine situations where, for a single-purpose project, the application of these cost-allocation theories would lead to inequitable and discriminative results. There are many cases in which the cost of obtaining a given service in one way has no conceivable relation to the cost of obtaining it in another way; for example, the customers of a water utility may be able to supply their own requirements privately at varying unit costs, depending on their respective circumstances. There is no basis for assuming that the cost to the utility to supply these various customers would bear any relation to the costs to such users to supply themselves. Since the application of the alternative cost or of any other method may lead to arbitrary and discriminative results when applied to such a relatively simple case, it follows that the same result may be obtained when these same methods

NOTE.—This paper by Theodore B. Parker, M. Am. Soc. C. E., was published in December, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1942, by Malcolm Elliott, M. Am. Soc. C. E.; March, 1942, by E. L. Chandler, M. Am. Soc. C. E.; May, 1942, by Eugene L. Grant, M. Am. Soc. C. E.; and June, 1942, by Messrs. Sherman M. Woodward, and Albert R. Arledge.

[12] Efficiency Engr., Pennsylvania Water & Power Co., Baltimore, Md.

[13] The late Pres., Safe Harbor Water Power Corp., Baltimore, Md. Mr. Allner died on July 18, 1942.

[13a] Received by the Secretary August 25, 1942.

are applied to the more complicated problem of allocating cost for multiple-purpose projects. As stated by the author, however, the more generally accepted methods for allocating the cost of a single-purpose project scarcely would be adaptable to a multiple-purpose project because of the absence of common characteristics of the services of the various purposes of such projects.

Even for electric utilities, there is no one generally accepted method for allocating cost to the various classes of customers. The reason that cost allocations are made in such cases is to determine the reasonableness of the charges made or to obtain a basis for determining what charges should be made. Regardless of the allocation of cost and the resulting charges for services, the total annual revenue to be received by the utility would depend only on the total annual cost of rendering service, provided, of course, that such cost is less than the cost of an alternative supply.

The justification of a multiple-purpose project, from an engineering point of view, depends only on the total annual cost of the services from the project being less than the minimum total annual justifiable cost (that is, cost which is less than the annual benefits) of obtaining the same services otherwise. The total annual revenue, as for a single-purpose project, should depend only on the total annual cost of rendering the services, provided it is not greater than the minimum annual cost of obtaining the same services otherwise. Except for this limitation, the total annual revenue should be independent of both the cost of obtaining the same services otherwise and of any cost allocation that may be made.

The only reason for making a cost allocation is to obtain a basis for the rates and charges for various services that will result in a total annual revenue as nearly equal to the total annual cost as possible. If it is the public policy not to collect revenue for various services of government-owned, multiple-purpose projects and to make only nominal charges for others, the resulting deficiencies will have to be subsidized from general taxes. The amount of such subsidies should be determined directly as a matter of public policy and not indirectly (as is most often done), by means of a cost allocation.

The questions as to whether or not a project should be subsidized and the extent of the subsidy are political questions rather than problems of engineering economics. The application of engineering economics to a project as a whole ends when it has been determined whether or not the project is economically justified; and the political decisions with respect to such projects, rather than the engineering and economic facts, should be based on the declared public policy. The answer to the question as to what extent, if any, it is the public policy to subsidize such projects depends in turn on the public's knowing the true annual cost, including the controversial question as to the "cost of money."

The money invested belongs to the public regardless of whether the project is "privately" or "publicly" owned; in the one case the money invested belongs to specific individuals or groups and in the other to the public generally. The economics of a project do not change merely because it is sponsored by the government. An important difference between the financing of a project by the government or by a private company is that the government, particularly the federal government, has much greater resources. An apparent difference

arises from the fact that, because of tax regulations and the safety to investors in government securities provided by its taxing powers, the government is usually able to borrow at a lower rate than a private enterprise. This does not necessarily mean, as is often assumed, that the cost of money to the government for any particular purpose is less than that to a responsible private company for the same kind of undertaking. This is true even though the government, if it so desires, may finance a project entirely with borrowed money, whereas private companies must usually provide substantial portions of equity money in addition to that borrowed.

It is frequently forgotten that the government's "equity" is provided by the taxpayers who must bear the burden of all losses suffered on account of investments in such projects. It is difficult to understand logic that assumes the public places less value on its money and is willing for the money to earn a lower return simply because it was invested through the government instead of a private company. The answer is that the money used in the same project is worth the same, regardless of who made the investment, assuming equally capable management in each case, since the risks would then be the same. Further, it is not believed that the interest rate paid by the government alone tells the entire story as to the cost of the public's money used by the government for such purposes. The present situation as to taxes makes this point more clear. The unevaluated portion of the cost of the public's money used by the government is the necessary sacrifice of the public to make money available to the government for its various activities. This fact has always existed and is only made more clear by present conditions. For example, if one assumes no public debt (as a result of a pay-as-you-go program), the government would pay no interest on the money expended by it for any purpose. Should one then assume zero cost for the money spent by the government for such projects? The fact of the matter is that there is no magic in government ownership, and the cost of money to it is the average return demanded by the public itself for investments in reasonably safe private undertakings. Such a basis for determination serves as an indirect, but nevertheless reliable, method of evaluating previously neglected elements of cost of money to the government. The economic justification of the project and its true annual total cost should both be computed on the foregoing basis for determining the cost of money. The total annual revenue should be equal to this total annual cost to make the project self-supporting. However, it may be the public policy that no revenue be received from some of the purposes of the project, with only nominal revenue from others, and normal revenue from the remaining purposes.

To illustrate, assume a multiple-purpose project built for flood control, navigation, power and energy production, and recreation; and assume that it is the public policy that no direct charges be made for flood-protection benefits, that only nominal charges be made for use of the recreation facilities, and that normal charges be made for navigation and power and energy. The total annual cost of the project is \$3,700,000, of which \$300,000 is the annual cost incurred specifically for flood control, \$400,000 specifically for navigation, \$1,500,000 specifically for power and energy production, \$100,000 specifically for recreation, and the remainder of \$1,400,000 for joint facilities. The nominal

charges made for the use of the recreation facilities result in an annual revenue of $20,000. The problem is to determine the rates and charges that should be made for the use of the navigation facilities and for power and energy. It is believed, as stated previously, that there is no correct engineering solution to this problem, in the scientific sense, and that the determination of the cost allocation to the various purposes is necessarily arbitrary, regardless of how it may be derived.

The total annual revenue desired is likewise a matter of public policy, which one may assume would be either (1) as large as possible, (2) approximately equal to the total annual cost of the project even though flood control is to be wholly, and recreation largely, subsidized, (3) equal to some specified amount, (4) less than the total annual cost of the project by some specified amount, or (5) equal to some specified percentage of the total annual cost of the project.

It is believed that the underlying principle of the vendibility theory of cost allocation (namely, that services and commodities jointly produced are mutual by-products of each other and that the actual total annual revenue should be as nearly as possible equal to the desired total annual revenue) is the most logical basis for arriving at an equitable, but admittedly arbitrary, basis for allocating the cost of a government-owned, multiple-purpose project to its various purposes. The total revenue received on account of any of the purposes of the project is immaterial, so long as it is not greater than that required to defray the annual cost for the most economical means of obtaining the same services otherwise.

The charges for each of the services should be equal to or less than the smaller of (1) the economic rate (that is, the rate that would produce the maximum net income), and (2) the rate required to be paid for the most economical available means of obtaining the same services otherwise.

If the total annual revenue resulting from the predetermined nominal rates for certain services and those determined in the foregoing manner for the other services (which may be called either separately or in total the unadjusted revenue) is greater than the total desired for the project (as determined by public policy) the rates should be further reduced so that the desired over-all results will be obtained. Such an excess of revenue may result from various combinations of different amounts of earnings on account of the various purposes of the project. Whatever the combination is that causes this excess, it is suggested that the reduction be based on comparisons of the unadjusted annual revenue with the annual cost of the specific facilities for each of the purposes of the project, and of the total unadjusted annual revenue with the total revenue desired.

The simplest case results when the excess, if any, of unadjusted total annual revenue over the desired total annual revenue is derived from only one purpose of the project. This will be the case when there is only one purpose for which the unadjusted annual revenue exceeds the annual cost of specific facilities and this excess is greater than the amount the unadjusted total annual revenue exceeds the desired total annual revenue. It would appear illogical, under such circumstances, to further reduce the annual revenues for those purposes for which the unadjusted annual revenues fail to cover the annual cost of

specific facilities; and consequently, such reduction should all be applied to the one purpose for which the unadjusted annual revenue exceeds the annual cost of specific facilities. However, in the event (even though there is only one purpose for which the unadjusted annual revenue exceeds the annual cost of specific facilities for that purpose) that the excess of the unadjusted total annual revenue over the desired total annual revenue is greater than such excess for that one purpose, the situation becomes rather complex and more complicated treatment of the problem, as discussed later, is necessary.

If there are several purposes of the project for which the unadjusted annual revenues exceed the annual cost of specific facilities, there are, as brought out in the preceding special case, two possible conditions, one of which is relatively simple and the other more complex. The simple condition results when the excess of the unadjusted total annual revenue over the total annual revenue desired is less than the total of the excesses of the unadjusted annual revenues over the annual cost of specific facilities for those purposes of the project for which there are such excesses. The reduction to be made for the simple case should be on a proportional basis to those purposes of the project having such excess revenues.

To illustrate, assume for the example given previously that the unadjusted annual revenue is \$1,500,000 for navigation and \$2,800,000 for power and energy, which for navigation is \$1,100,000 and for power and energy \$1,300,000 in excess of the annual cost of the specific facilities for those purposes. These unadjusted annual revenues, together with those from recreation, give \$4,320,000 as the total annual revenue from the project. If the desired annual revenue is \$3,000,000, the total reduction for navigation and power and energy would be \$1,320,000, whereas the sum of the excess revenues for these purposes is \$2,400,000. The reduction to be applied to navigation then would be $\frac{11}{24} \times \$1{,}320{,}000$, or \$605,000, and to power and energy $\frac{13}{24} \times \$1{,}320{,}000$, or \$715,000. The resulting adjusted revenue for navigation would be \$895,000, and for power and energy \$2,085,000.

The more complex situations result when the desired total annual revenue of the project is so low, in comparison with the unadjusted total annual revenue, that the necessary reduction is greater than the total excess of unadjusted revenue over the annual cost of specific facilities for those purposes of the project for which such excess revenues exist. (The case in which there was no purpose of the project for which the unadjusted annual revenue exceeded the annual cost of specific facilities would also be included here.) In order to make the desired reduction in total revenue for such cases, not only revenues from otherwise "profitable" purposes of the project, if any, must be reduced below the annual costs of specific facilities for these purposes, but revenues from the "unprofitable" purposes of the project may also be further reduced.

The solution suggested herein involves the finding of a ratio, less than unity, which when applied to the specific cost of facilities for any purpose will give the adjusted revenue for that purpose, provided the revenue is reduced thereby (that is, the revenue for any purpose is to be reduced or must remain unchanged). The ratio to be determined must also satisfy the condition that ad-

justed revenues (including those that remain unchanged) must total to the desired revenue from the project.

To illustrate, assume the same data for the project as previously used, except that the total desired annual revenue is only $1,000,000 (that is, the subsidy has been increased $2,000,000 as compared with the previous example). The total annual cost of specific facilities, excluding those for flood control, is $2,000,000. This total multiplied by 50%, the ratio of the desired annual revenue to the total annual cost of specific facilities, would give the desired $1,000,000 total revenue. It is immediately apparent, however, that the 50% ratio applied to the cost of facilities for recreation ($100,000) would give a revenue in excess of the $20,000 predetermined for this purpose. Consequently, the computation of the ratio must be adjusted by excluding the annual cost of recreation facilities and recreation revenue. The total annual cost of specific facilities for navigation and power and energy is $1,900,000, and the desired revenue, after deducting $20,000 for recreation, $980,000. The ratio of desired revenue to cost is then 0.516. The adjusted revenue for navigation would then be $400,000 × 0.516, or $206,400, and for power and energy $1,500,000 × 0.516, or $773,600. The total adjusted revenue, including $20,000 from recreation, is the desired $1,000,000.

These two methods of distributing the excess of the unadjusted total annual revenue over the total annual revenue desired among the revenues from the various purposes of a multiple-purpose project have a common point at which their results necessarily agree. At this point the desired reduction in total revenue is just equal to the sum of the excesses of unadjusted annual revenues over the annual cost of specific facilities for those purposes for which excesses exist.

This suggested basis for determining the charges to be made for the various services rendered by multiple-purpose projects is not, in a strict sense, a basis for allocating cost. It is merely a basis for the determination of the charges to be made for these various services in accordance with a declared public policy, not only as to what purposes of the project are to be subsidized, but also as to what extent the project as a whole is to be subsidized. The procedure used by the author (assuming that the rates for the various services are based on the cost allocation) results in a subsidy for the project as a whole which varies as the cost allocated to the various purposes varies. The subsidy increases as the cost allocated to the no-revenue and small-revenue producing purposes of the project increases. On the other hand, the application of the foregoing suggested basis for determining the charges to be made for the various services of the project assures the return that is desired from the project as a whole in accordance with public policy, and the public policy as to what purposes shall be subsidized as no-revenue and small-revenue producing. At the same time it makes available the various services, for which normal charges are to be made, at the same or less cost than the minimum for which they could be obtained otherwise. Thus, the beneficiaries of such services have no basis for complaint, since they are not hurt, whereas the interests of the general public are protected to a definite predetermined extent.